MATHS GURUS 2020

EQUATION OF A CIRCLE, GRAPHS OF CUBIC FUNCTIONS & GRAPHICAL DETERMINATION OF LAWS

EQUATIONS OF A CIRCLE

1. Determine the equation of a circle centre $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and radius $r = 3$ units. Give your answer in the form $ax^2 + by^2 + cx + dy + e = 0$ where a, b, c, d and e are integers.

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = 9$$

$$x^2 - x + \frac{1}{4} + y^2 + y + \frac{1}{4} = 9$$

$$4x^2 - 4x + 1 + 4y^2 + 4y + 1 = 36$$

$4x^2 + 4y^2 - 4x + 4y - 34 = 0$ or

$$2x^2 + 2y^2 - 2x + 2y - 17 = 0$$

2. The points with coordinates $(7, -10)$ and $(-3, 8)$ are the end points of a diameter of a circle centre A. Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are constants.

Centre A $\Longrightarrow$ $mid\ point$ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{7-3}{2}, \frac{-10+8}{2}\right) \Longrightarrow A(2, -1)$$

$$Radius \Longrightarrow \sqrt{(2+3)^2 + (-1-8)^2} = \sqrt{106} = 10.30\ units$$

$$(x-2)^2 + (y+1)^2 = \left(\sqrt{106}\right)^2$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 106$$

$$x^2 + y^2 - 4x + 2y - 101 = 0$$

3. The equation of a circle is given as $4x^2 + 4y^2 - 16x + 24y + 3 = 0$. Find the centre of the circle and its radius.

$$x^2 + y^2 - 4x + 6y = -\frac{3}{4}$$

$$x^2 - 4x + \left(-\frac{4}{2}\right)^2 + y^2 + 6y + \left(\frac{6}{2}\right)^2 = -\frac{3}{4} + \left(-\frac{4}{2}\right)^2 + \left(\frac{6}{2}\right)^2$$

$$(x-2)^2 + (y+3)^2 = \frac{49}{4}$$

Centre $(2, -3)$ and radius $r = \frac{7}{2} = 3.5\ units$

4. The equation of a circle is given as $x^2 + y^2 + 4x - 5 = 0$. Find the centre of the circle and its radius.

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 + 0y + \left(\frac{0}{2}\right)^2 = 5 + \left(\frac{4}{2}\right)^2 + \left(\frac{0}{2}\right)^2$$

$$(x+2)^2 + (y+0)^2 = 9$$

Centre $(-2, 0)$ and radius $= 3\ units$

5. obtain the centre and the radius of the circle represented by equation $x^2 + y^2 - 10y + 16 = 0$.

   $$x^2 + 0x + \left(\frac{0}{2}\right)^2 + y^2 - 10y + \left(-\frac{10}{2}\right)^2 = -16 + \left(\frac{0}{2}\right)^2 + \left(-\frac{10}{2}\right)^2$$

   $(x + 0)^2 + (y - 5)^2 = 9$

   Centre $(0, 5)$ radius $= 3\ units$

6. Show that $3x^2 + 3y^2 + 6x - 12y - 12 = 0$ is an equation of a circle hence state the radius and the centre of the circle.

   $x^2 + y^2 + 2x - 4y - 4 = 0$

   $$x^2 + 2x + \left(\frac{2}{2}\right)^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 4 + \left(\frac{2}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

   $(x + 1)^2 + (y - 2)^2 = 9 \Leftrightarrow$ It is an equation of a circle

   Centre $(-1, 2)$ & radius $= 3\ units$

7. Find the centre and the radius of a circle whose equation is given by $4(x^2 + y^2) = 12(x - y) + 7$

   $4x^2 + 4y^2 - 12x + 12y = 7$

   $x^2 + y^2 - 3x + 3y = \frac{7}{4}$

   $$x^2 - 3x + \left(\frac{-3}{2}\right)^2 + y^2 + 3y + \left(\frac{3}{2}\right)^2 = \frac{7}{4} + \left(\frac{-3}{2}\right)^2 + \left(\frac{3}{2}\right)^2$$

   $$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$$

   Centre $(1.5, -1.5)$ and radius $= 2.5\ units$

8. Find the centre and the radius of a circle whose equation is given by $2x(x - 3) + 2y(y + 5) + 9 = 0$.

   $2x^2 - 6x + 2y^2 + 10y + 9 = 0$

   $x^2 - 3x + y^2 + 5y = \frac{9}{2}$

   $$x^2 - 3x + \left(\frac{-3}{2}\right)^2 + y^2 + 5y + \left(\frac{5}{2}\right)^2 = \frac{9}{2} + \left(\frac{-3}{2}\right)^2 + \left(\frac{5}{2}\right)^2$$

   $(x - 1.5)^2 + (y + 2.5)^2 = 4$

   Centre $(1.5, -2.5)$ and radius $= 2\ units$

9. AB is the diameter of a circle. Given the co – ordinates of A and B as $(2, -3)$ and $(4, -7)$respectively, find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$ where *a, b and c* are integers.

   Centre $\left(\frac{4+2}{2}, \frac{-7-3}{2}\right) \Rightarrow (3, -5)$

Radius $= \sqrt{(3-2)^2 + (-5+3)^2} = \sqrt{5}$

$(x-3)^2 + (y+5)^2 = \left(\sqrt{5}\right)^2$

$x^2 - 6x + 9 + y^2 + 10y + 25 = 5$

$x^2 + y^2 - 6x + 10y + 29 = 0$

10. Find the equation of a circle with the end points of a diameter at $(4, 3)$ and $(0, 1)$. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are integers.

Centre $\left(\frac{4+0}{2}, \frac{3+1}{2}\right) \Rightarrow (2, 2)$

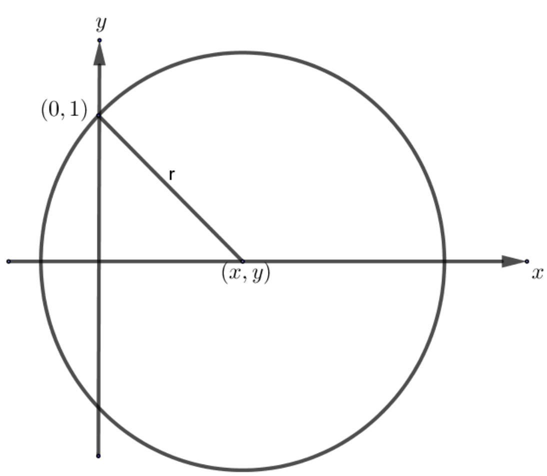
Radius $= \sqrt{(2-0)^2 + (2-1)^2} = \sqrt{5}$

$(x-2)^2 + (y-2)^2 = \left(\sqrt{5}\right)^2$

$x^2 - 4x + 4 + y^2 - 4y + 4 = 5$

$x^2 + y^2 - 4x - 4y + 3 = 0$

11. Find the circles that satisfy the conditions: Radius $= \sqrt{17}$, centre on the x – axis, and passes through the point (0,1) . Give your answers in the form in the form $x^2 + y^2 + ax + by + c = 0$ where , ab and c are integers.



Centre $(x, 0)$

$(x-0)^2 + (0-1)^2 = 17$

$x^2 + 1 = 17$

$x = \sqrt{16} = \pm 4$

Centres $(4, 0)$ *or* $(-4, 0)$

$(x-4)^2 + (y-0)^2 = 17$

$x^2 - 8x + 16 + y^2 = 17$

$x^2 + y^2 - 8x - 1 = 0$

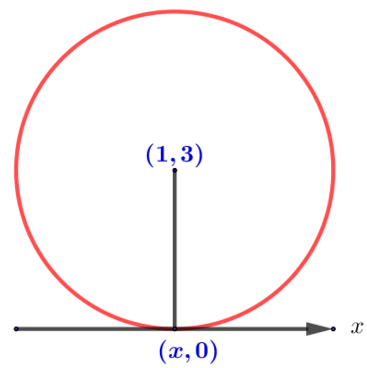
OR

$(x+4)^2 + (y-0)^2 = 17$

$x^2 + 8x + 16 + y^2 = 17$

$x^2 + y^2 + 8x - 1 = 0$

12. A circle whose centre is at $(1, 3)$ has the x – axis as its tangent. Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are integers.



$(x-1)^2+(y-3)^2=r^2$

$r^2=(3-0)^2=9$

$(x-1)^2+(y-3)^2=9$

$x^2-2x+1+y^2-6y+9=9$

$x^2+y^2-2x-6y+1=0$

13. Find the equation of the line containing the centres of the two circles $x^2+y^2+6x+4y+9=0$ and $x^2+y^2-4x+6y+4=0$.

i. $x^2-4x+\left(\frac{-4}{2}\right)^2+y^2+6y+\left(\frac{6}{2}\right)^2=-4+\left(\frac{-4}{2}\right)^2+\left(\frac{6}{2}\right)^2$

$(x-2)^2+(y+3)^2=9$

Centre $(2,-3)$ radius $=3\ units$

ii. $x^2+6x+\left(\frac{6}{2}\right)^2+y^2+4y+\left(\frac{4}{2}\right)^2=-9+\left(\frac{6}{2}\right)^2+\left(\frac{4}{2}\right)^2$

$(x+3)^2+(y+2)^2=4$

Centre $(-3,-2)$ radius $=2\ units$

- End points of the line are $(2,-3)$ and $(-3,-2)$

$m=\frac{-2+3}{-3-2}=\frac{1}{-5}$

$\frac{y+3}{x-2}=\frac{1}{-5}$

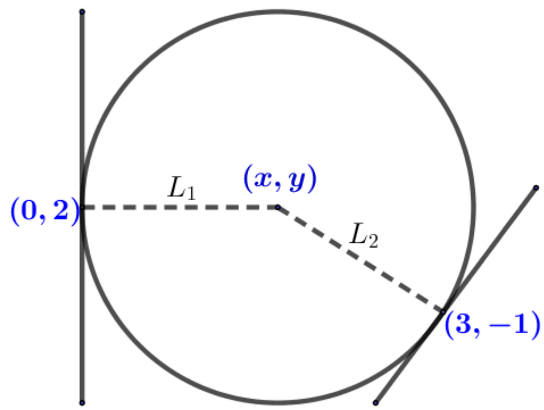
$-5(y+3)=1(x-2)$

$-5y-15=x-2$

$x+5y+13=0$

14. The line $x-2y+4=0$ is a tangent to a circle at $(0,2)$. The line $y=2x-7$ is a tangent to the same circle at $(3,-1)$. Find:

a) The centre and radius of the circle.



Equation of $L_1 \Longrightarrow \frac{y-2}{x-0} = -2$

$\Longrightarrow y = -2x + 2$

Equation of $L_2 \Longrightarrow \frac{y+1}{x-3} = \frac{-1}{2}$

$\Longrightarrow y = -\frac{1}{2}x + \frac{1}{2}$

$\therefore -2x + 2 = -\frac{1}{2}x + \frac{1}{2}$

$4 - 4x = -x + 1$

$3 = 3x$

$\Leftrightarrow x = 1 \text{ and } y = 0$

Centre $(1, 0)$

Radius $= \sqrt{(3-1)^2 + (-1-0)^2} = \sqrt{5}$

b) The equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c $are\ integers$.

$(x-1)^2 + (y-0)^2 = (\sqrt{5})^2$

$x^2 + y^2 - 2x + 1 = 5$

$x^2 + y^2 - 2x - 4 = 0$

15. The area of an annulus between two concentric circles is8π square units. If the equation of the larger circle is given as $4x^2 + 4y^2 - 8x + 8y - 73 = 0$, find the equation of the smaller circle.

Larger circle

$4x^2 + 4y^2 - 8x + 8y = 73$

$x^2 + y^2 - 2x + 2y = \frac{73}{4}$

$x^2 - 2x + \left(\frac{-2}{2}\right)^2 y^2 + 2y + \left(\frac{2}{2}\right)^2$

$= \frac{73}{4} + \left(\frac{-2}{2}\right)^2 + \left(\frac{2}{2}\right)^2$

$(x-1)^2 + (y+1)^2 = \frac{81}{4}$

Centre $(1, -1)$ radius $= \frac{9}{2} = 4.5$

$\pi R^2 - \pi r^2 = 8\pi$

$\pi\left(\frac{9}{2}\right)^2 - \pi r^2 = 8\pi$

$\frac{81}{4}\pi - \pi r^2 = 8\pi$

$81\pi - 4\pi r^2 = 32\pi$

$\frac{49}{4} = r^2$

$r = \frac{7}{2} = 3.5$

Equation of the smaller circle

$(x-1)^2 + (y+1)^2 = \frac{49}{4}$

$x^2 - 2x + 1 + y^2 + 2y + 1 = \frac{49}{4}$

$4x^2 - 8x + 4 + 4y^2 + 8y + 4 = 49$

$4x^2 + 4y^2 - 8x + 8y - 41 = 0$

16. Calculate the length of the tangent from a point $(-9, 9)$ to the circle whose equation is $x^2 + y^2 + 6x - 10y - 2 = 0$.

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 = 2 + \left(\frac{6}{2}\right)^2 + \left(\frac{-10}{2}\right)^2$$

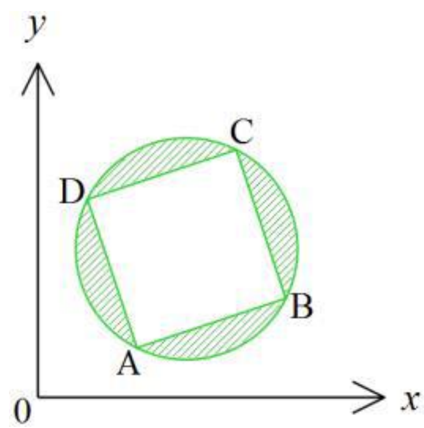
$(x + 3)^2 + (y - 5)^2 = 36$

Centre $(-3, 5)$ Radius $= 6$ units

$AO = \sqrt{(-9 + 3)^2 + (9 - 5)^2} = \sqrt{36 + 16} = \sqrt{52}$

$AB = \sqrt{\left(\sqrt{52}\right)^2 - 6^2} = \sqrt{52 - 36} = 4\ units$

17. In the figure below, ABCD is a square inscribed in a circle whose equation is $x^2 + y^2 - 6x - 6y + 10 = 0$. Calculate, in terms of π, the area of the shaded segments.



$$x^2 - 6x + \left(\frac{-6}{2}\right)^2 + y^2 - 6y + \left(\frac{-6}{2}\right)^2 = -10 + \left(\frac{-6}{2}\right)^2 + \left(\frac{-6}{2}\right)^2$$

$(x - 3)^2 + (y - 3)^2 = 8$

Centre $(3, 3)$ Radius $= \sqrt{8}$

Area of the circle $= \pi r^2 = \pi\left(\sqrt{8}\right)^2 = 8\pi$

Considering the square $\Longrightarrow l^2 + l^2 = \left(2\sqrt{8}\right)^2$

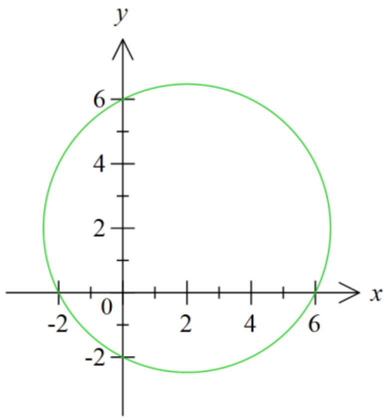
$2l^2 = 4 \times 8$

$l = \sqrt{16} = 4\ units$

Area of the square $= 4 \times 4 = 16$

Area of shaded region $= (8\pi - 16)$ sq. units

18. In the figure below, the circle passes through the points $(-2, 0)$, $(6, 0)$, $(0, -2)$ and $(0, 6)$.



Find:

(a) The centre and the radius of the circle.

$y \Longrightarrow \frac{6-2}{2} = 2$

$x \Longrightarrow \frac{6-2}{2} = 2$

Centre $(2, 2)$

$r = \sqrt{(2-0)^2 + (2-6)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

(b) The equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$ where a, $b$and c are integers

$(x-2)^2 + (y-2)^2 = \left(\sqrt{20}\right)^2$

$x^2 - 4x + 4 + y^2 - 4y + 4 = 20$

$x^2 + y^2 - 4x - 4y - 12 = 0$

19. Given that a circle whose equation is $0.25x^2 + 0.25y^2 + x + ky - 1 = 0$ passes through a point $(-2, 4)$. Find:

(a) The value of k

$0.25x^2 + 0.25y^2 + x + ky - 1 = 0$

$\frac{1}{4}x^2 + x + \frac{1}{4}y^2 + ky = 1$

$x^2 + 4x + y^2 + 4ky = 4$

$x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 + 4ky + \left(\frac{4k}{2}\right)^2 = 4 + \left(\frac{4}{2}\right)^2 + \left(\frac{4k}{2}\right)^2$

$(x+2)^2 + (y+2k)^2 = 8 + 4k^2$

Centre $(-2, -2k)$ radius = $\sqrt{8+4k^2}$

$r = \sqrt{(-2+2)^2 + (-2k-4)^2} = \sqrt{8+4k^2}$

$4k^2 + 16k + 16 = 8 + 4k^2$

$16k = -8$

$k = -\frac{1}{2}$

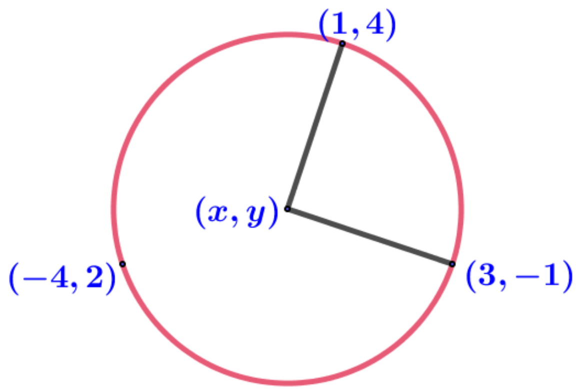
(b) The centre and the radius of the circle.

Centre $\left[\left(-2,\left(-2\times\frac{-1}{2}\right)\right)\right]=(-2,1)$

Radius $=\sqrt{8+\left(4\times\frac{1}{4}\right)}=\sqrt{9}=3\ units$

20. Three points A$(3,-1)$, B$(1,4)$ and C$(-4,2)$ lie on a circle. Calculate:

(a) The centre and the radius of the circle.



$(x-1)^2+(y-4)^2=(x-3)^2+(y+1)^2$

$4x-10y=-7$ ...... (i)

$(x-1)^2+(y-4)^2=(x+4)^2+(y-2)^2$

$10x+4y=-3$ ...... (ii)

Solving eqn (i) and (ii) simultaneously,

$20x+8y=-6$

$\underline{20x-50y=-35\ -}$

$58y=29$

$\Longrightarrow y=\frac{1}{2}$

$4x=-7+\left(10\times\frac{1}{2}\right)=-2$

$\Longrightarrow x=-\frac{1}{2}$

Centre $\left(-\frac{1}{2},\frac{1}{2}\right)$

$r=\sqrt{\left(1+\frac{1}{2}\right)^2+\left(4-\frac{1}{2}\right)^2}$

$=\sqrt{2.25+12.25}$

$=\sqrt{14.5}=3.808$

(b) The equation of the circle in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\left(\sqrt{14.5}\right)^2$

$x^2+x+\frac{1}{4}+y^2-y+\frac{1}{4}=14.5$

$x^2+y^2+x-y-14=0$

GRAPHS OF CUBIC FUNCTIONS

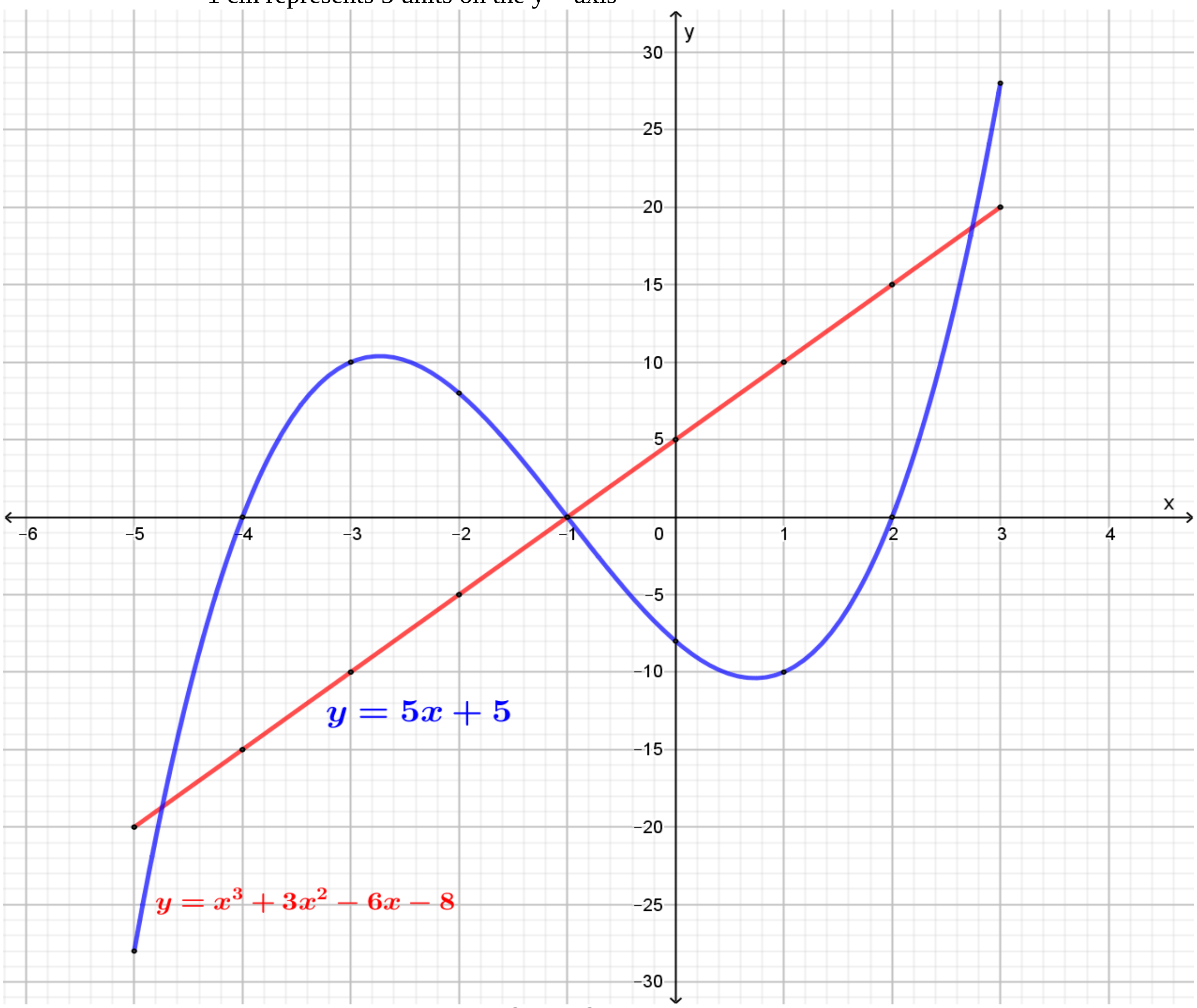
1. (a) Complete the table below for $y = x^3 + 3x^2 - 6x - 8$

x	-5	-4	-3	-2	-1	0	1	2	3
y	-28	0	10	8	0	-8	-10	0	28

(b) On the grid provided, draw the graph of $y = x^3 + 3x^2 - 6x - 8$ for $-5 \leq x \leq 3$

Use the scale: 1 cm represents 1 unit on the x – axis

1 cm represents 5 units on the y – axis



(c) (i) Use the graph to solve the equation $x^3 + 3x^2 - 6x - 8 = 0$

Values of x are $x = -4, x = -1$ and $x = 2$

(ii) By drawing a suitable straight line on the graph, solve the equation

$x^3 + 3x^2 - 6x - 8 = 5x + 5$

Drawing the graph of the line $y = 5x + 5$ for $-5 \leq x \leq 3$

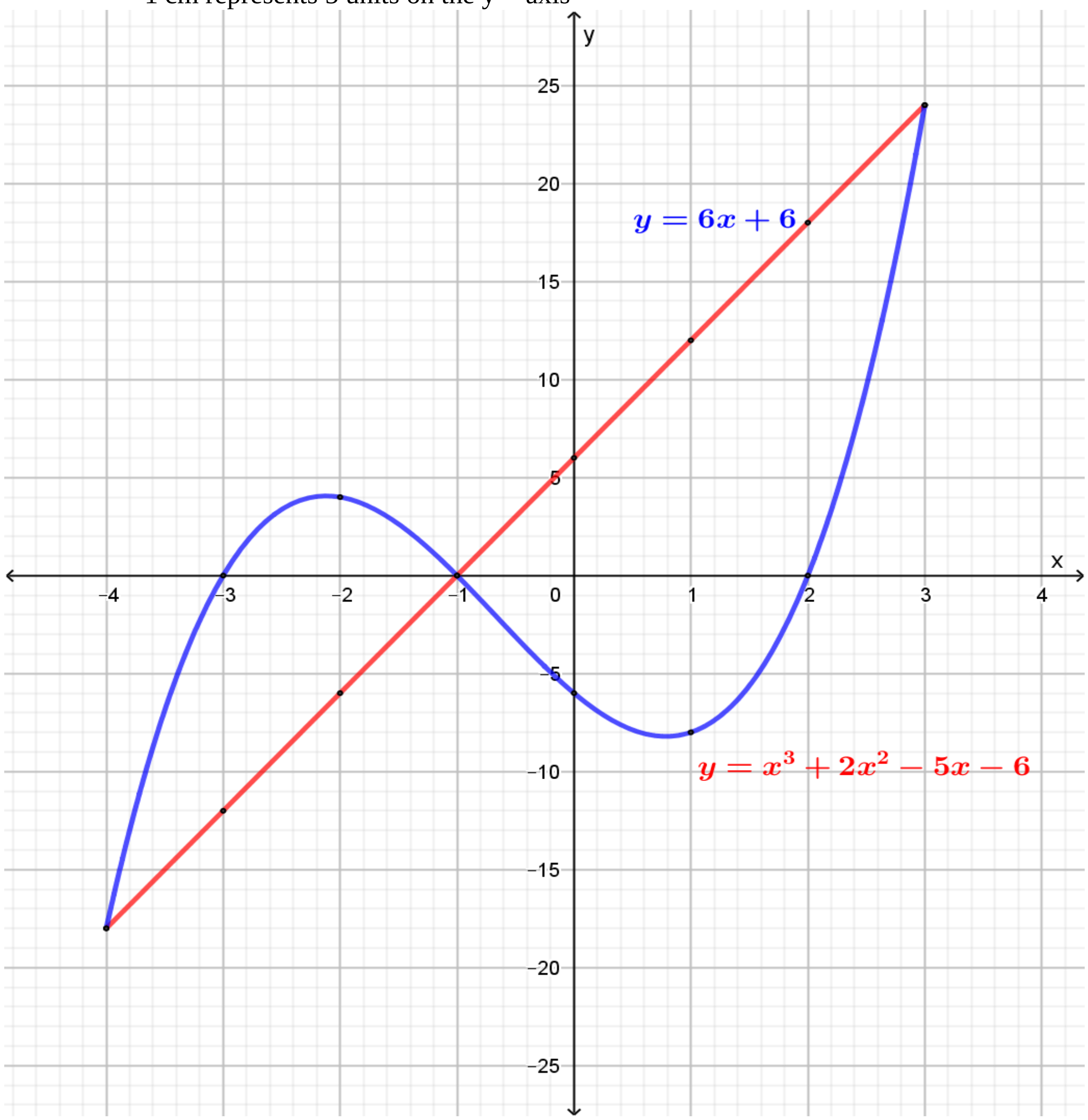
x	-5	-4	-3	-2	-1	0	1	2	3
y	-20	-15	-10	-5	0	5	10	15	20

Values of x are $x = -4.74, x = -1$ and $x = 2.74$

2. (a) Complete the table below for $y = x^3 + 2x^2 - 5x - 6$

x	-4	-3	-2	-1	0	1	2	3
y	-18	0	4	0	-6	-8	0	24

(b) On the grid provided, draw the graph of $y = x^3 + 2x^2 - 5x - 6$ for $-4 \leq x \leq 3$
Use the scale: 1 cm represents 1 unit on the x – axis
1 cm represents 5 units on the y – axis



(c) (i) Use the graph to solve the equation $x^3 + 2x^2 - 5x - 6 = 0$

Values of x are $x = -3, x = -1$ *and* $x = 2$

(ii) By drawing a suitable straight line on the graph, solve the equation $x^3 + 2x^2 - 11x - 12 = 0$

Drawing the graph of the line $y = 6x + 6$

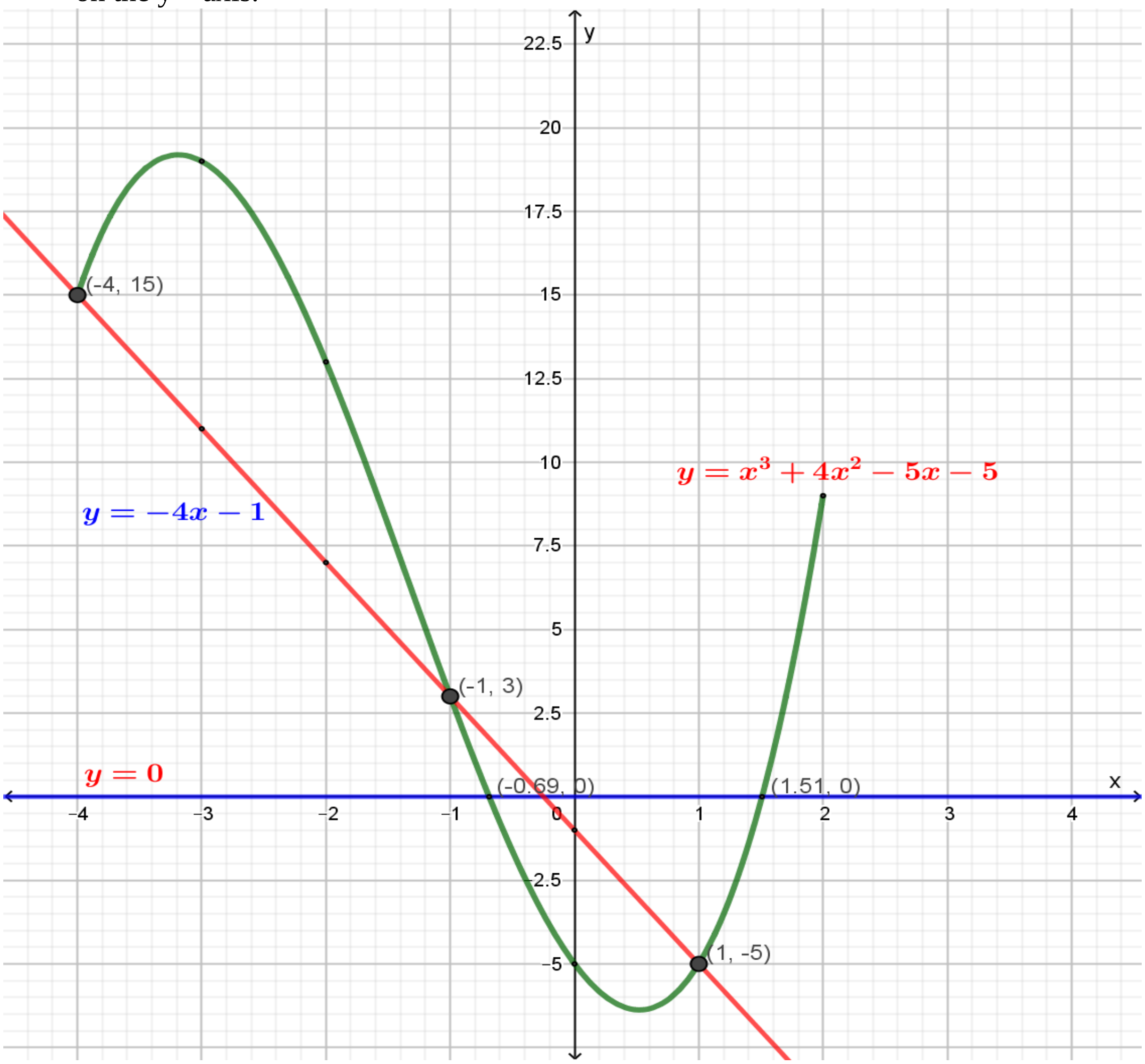
x	-4	-3	-2	-1	0	1	2	3
y	-18	-12	-6	0	6	12	18	24

Values of x are $x = 3, x = -1$ *and* $x = -4$

3. (a) Complete the table below for $y = x^3 + 4x^2 - 5x - 5$ for $-5 \leq x \leq 2$

x	-4	-3	-2	-1	0	1	2
y	15	19	13	3	-5	-5	9

(b) On the grid provided draw the graph of for $y = x^3 + 4x^2 - 5x - 5$ for $-5 \le x \le 2$
Use a scale of 1 cm to represent 1 unit on the x – axis and 2 cm to represent 5 units on the y – axis.



(c) Use your graph to solve the equations;

i. $x^3 + 4x^2 - 5x - 5 = 0$

Values of x are $x = 1.51 and\ x = -0.69$

ii. $x^3 + 4x^2 - 5x - 5 = -4x - 1$

Drawing the graph of $y = -4x - 1\ for - 4 \le x \le 2$

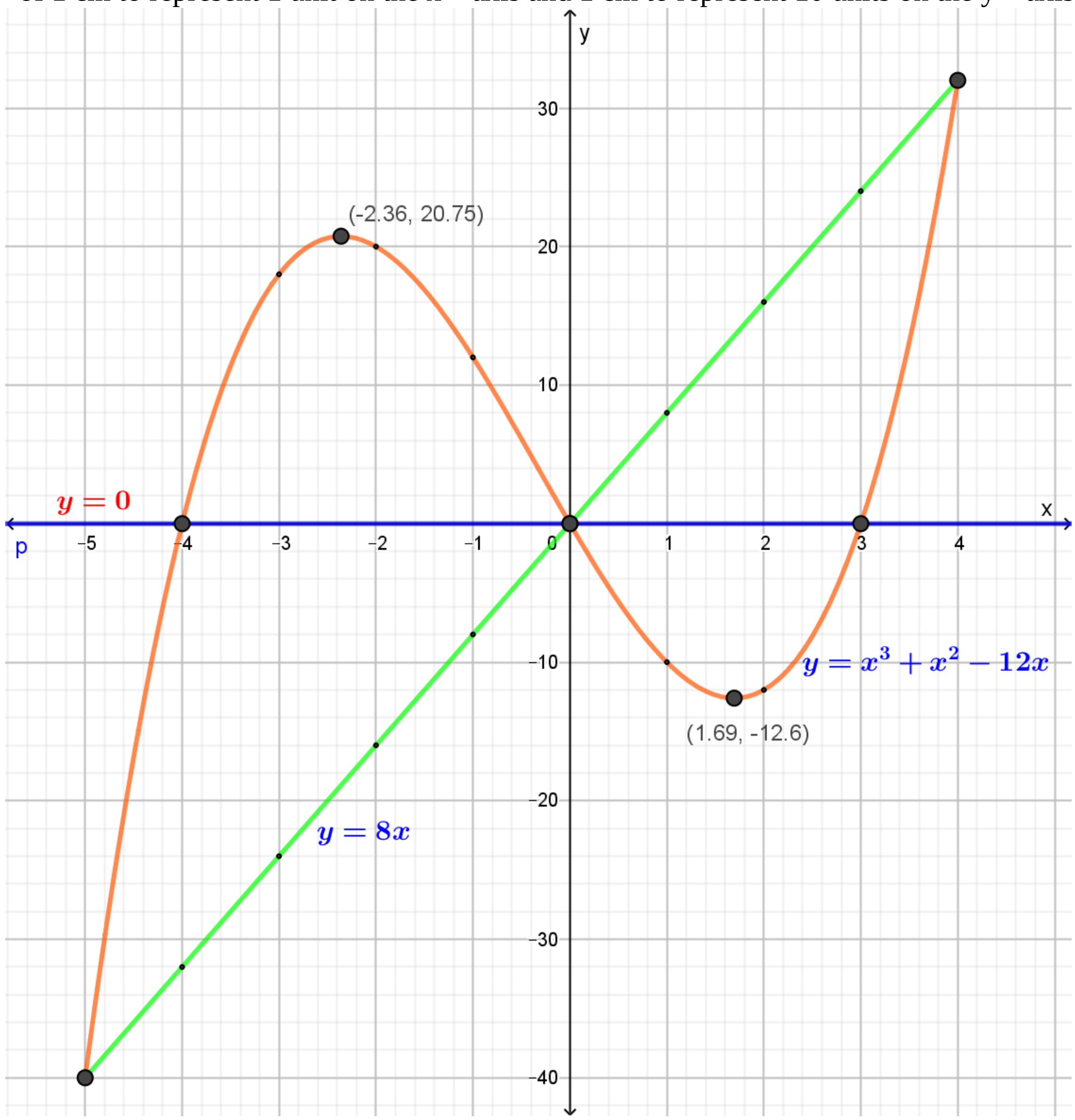
x	-4	-3	-2	-1	0	1	2
y	15	11	7	3	-1	-5	-9

Values of x are $x = 1, x = -1\ and\ x = -4$

4. (a) Complete the table below for $y = x^3 + x^2 - 12x$ for $-5 \le x \le 4$

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	-40	0	18	20	12	0	-10	-12	0	32

(b) On the grid provided draw the graph of for $y = x^3 + x^2 - 12x$ for $-5 \leq x \leq 4$. Use a scale of 1 cm to represent 1 unit on the x – axis and 1 cm to represent 10 units on the y – axis.



(c) Use your graph to solve the equations;

i. $x^3 + x^2 - 12x = 0$

Values of x are $x = 0, x = 3$ and $x = -4$

ii. $x^3 + x^2 - 20x = 0$

Drawing the graph of $y = 8x$ $for -5 \leq x \leq 4$

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	-40	-32	-24	-16	-8	0	8	16	24	32

Values of x are $x = 4, x = 0$ and $x = -5$

(d) State the maximum and minimum value of $y = x^3 + x^2 - 12x$

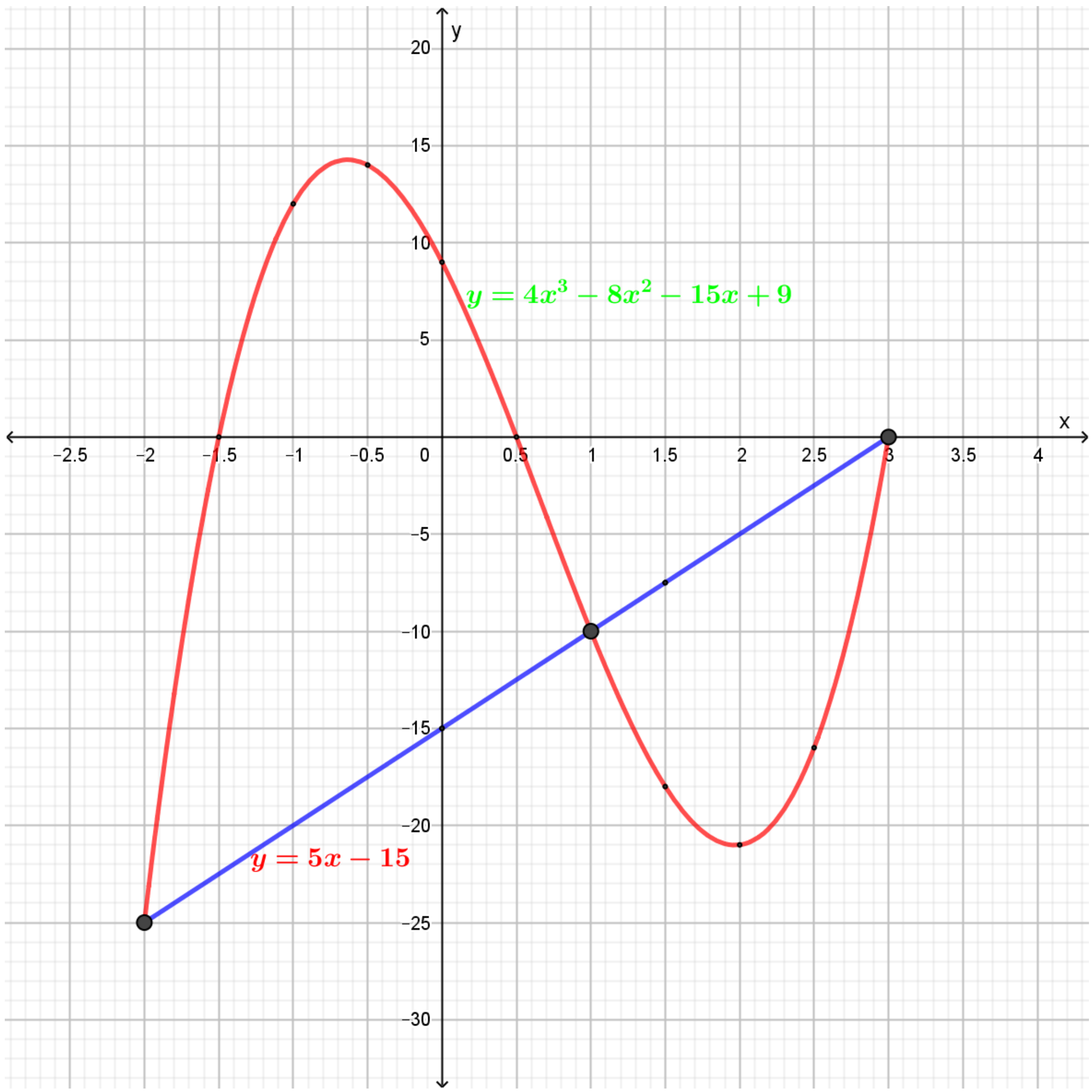
Maximum turning point $(-2.36, 20.75)$

Minimum turning point $(1.69, -12.6)$

5. (a) Complete the table below for $y = 4x^3 - 8x^2 - 15x + 9$ for $-2 \leq x \leq 3$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	-25	0	12	14	9	0	-10	-18	-21	-16	0

(b) On the grid provided draw the graph of for $y = 4x^3 - 8x^2 - 15x + 9$ for $-2 \leq x \leq 3$. Use a scale of 2 cm to represent 1 unit on the x – axis and 1 cm to represent 5 units on the y – axis.



(c) Using the graph drawn;

i. Solve the equation $4x^3 - 8x^2 - 15x + 9 = 5x - 15$

Drawing the graph of $y = 5x - 15$

x	-2	0	1.5	3
y	-25	-15	-7.5	0

Values of x are $x = 3, x = 1$ and $x = -2$

ii. Find the range of values of x for which:

$4x^3 - 8x^2 - 15x + 9 \geq 0$

Values of x are $-1.5 \leq x \leq 0.5$

$4x^3 - 8x^2 - 15x + 9 < 0$

Values of x are $-\infty \leq x < -1.5$ and $0.5 \leq x \leq 3$

<u>**GRAPHICAL DETERMINETION OF LAWS**</u>

1. Two variables A and B are connected by the equation $A = kB^n$ where k and n are constants. The table below gives values of A and B.

A	1.5	1.95	2.51	3.20	4.50
B	1.59	2.51	3.98	6.31	11.5

a) Find a linear equation connecting A and B. (2 marks)

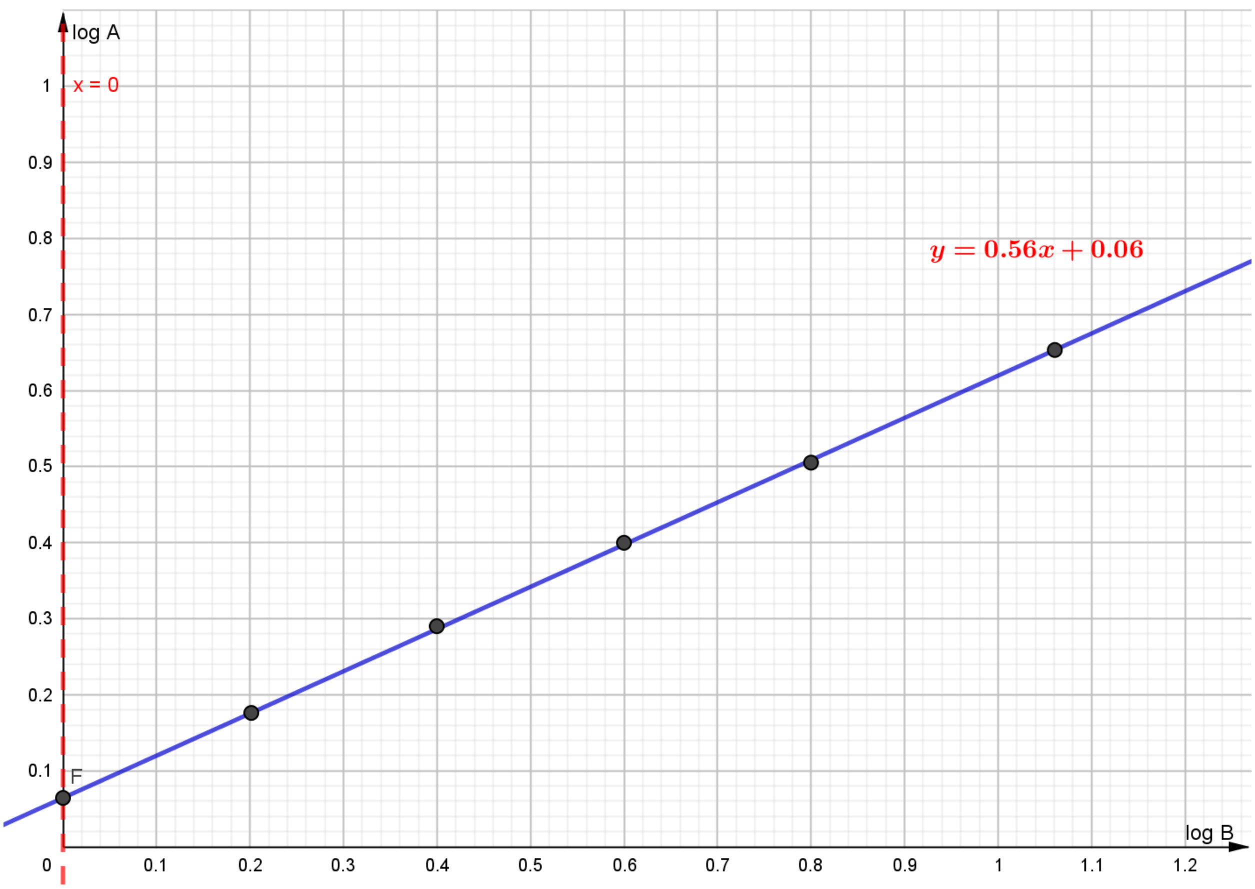
$A = kB^n$

Introducing logarithm to base 10 on both sides, we get:

$\log A = \log k + n \log B$

b) On square paper draw a suitable line graph to represent the relation in (a) above (scale 1cm to represent 0.1 units on both axis). (5 marks)

$\log A$	0.1761	0.2900	0.3997	0.5051	0.6532
$\log B$	0.2014	0.3997	0.5999	0.8000	1.0607



c) Use your graph to estimate the values of k and n in to one decimal place. (3 marks)

$\log k = 0.06$

$k = 10^{0.06} = 1.148$

$n = gradient\ of\ the\ graph = 0.56$

2. Two quantities P and n, are connected by the equation $P = AK^n$ where A and K are constants. The table below shows some corresponding values of n and P.

N	2	4	6	8	10
P	9.8	19.4	37.4	74.0	144.0

a) State the linear equation connecting P and n. (2 marks)

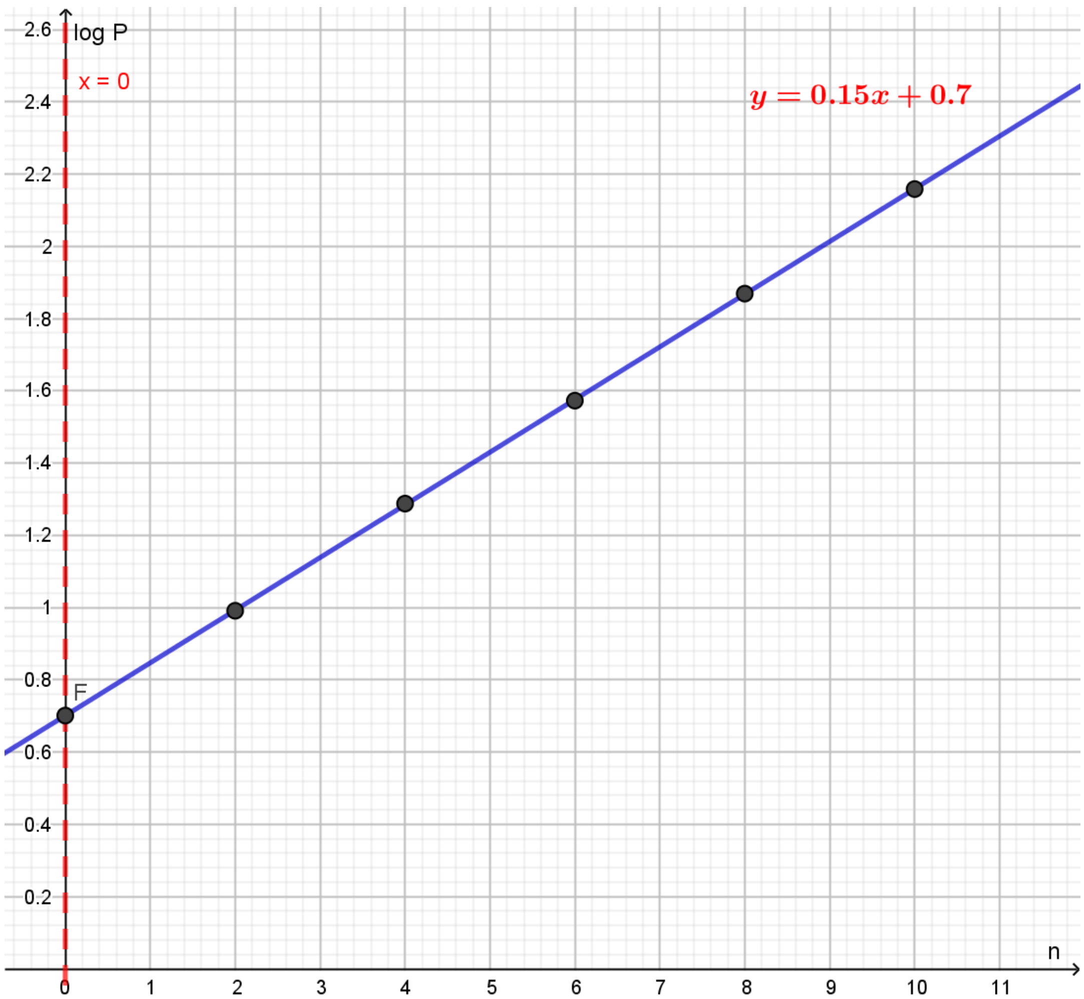
$P = AK^n$

Introducing logarithms on both sides, we get:

$\log P = \log A + n\log K$

b) On the grid provided, draw a suitable straight line. (5 marks)

N	2	4	6	8	10
P	9.8	19.4	37.4	74.0	144.0
Log P	0.9912	1.2878	1.5729	1.8692	2.1584



c) Use your graph to estimate the value of A and k. (3 marks)

$\log A = 0.7$

$A = 10^{0.7} = 5.012$

$\log k = 0.15$

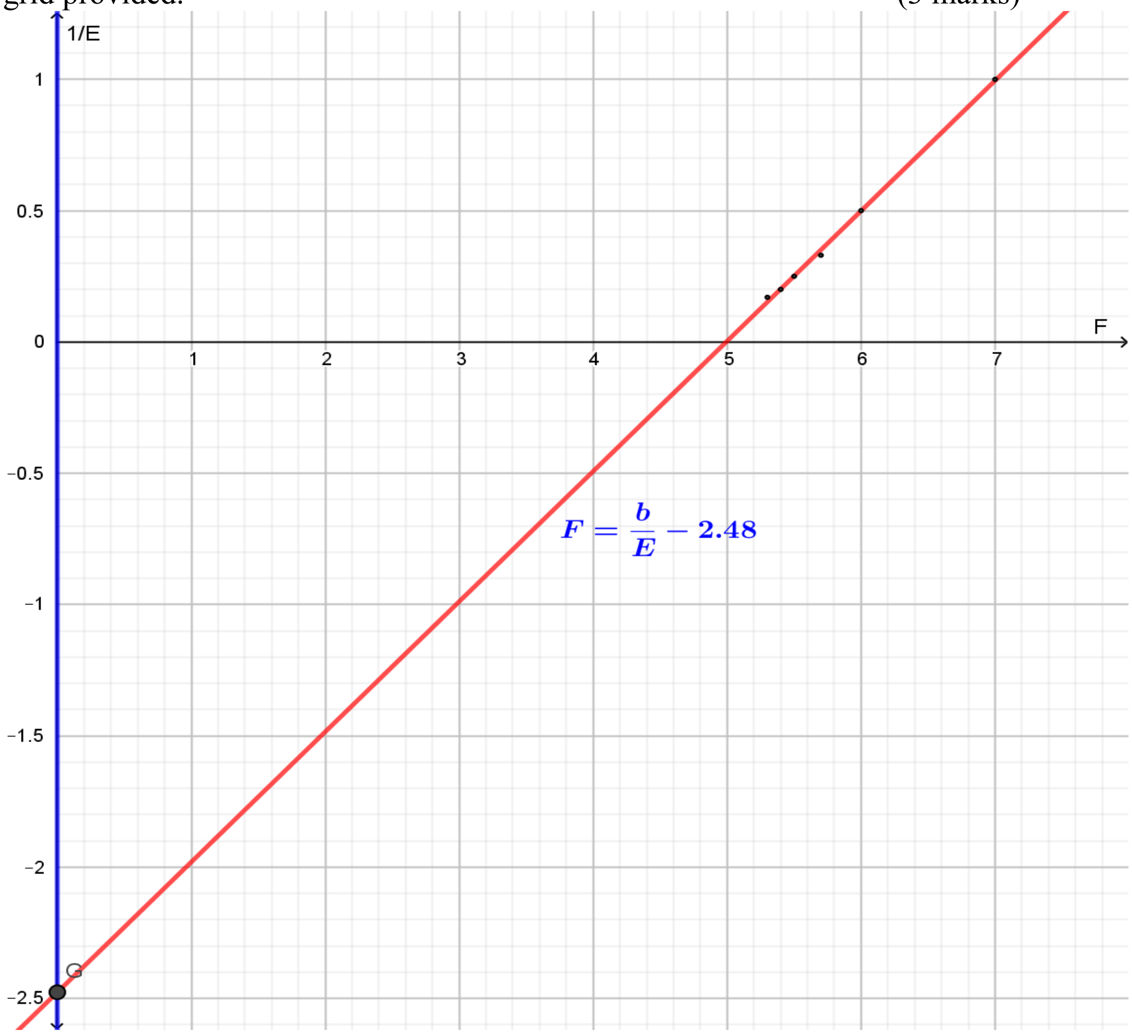
$k = 10^{0.15} = 1.4125$

4. The relationship between the two variables E and F is believed to be of the form $F = a + bE^{-1}$, where **a** and **b** are constants.

a) Complete the table below to two decimal places. (2 marks)

E	1	2	3	4	5	6
F	7.0	6.0	5.7	5.5	5.4	5.3
$^1/_E$	1	0.50	0.33	0.25	0.20	0.17

b) Use the values on the table above to draw a suitable linear graph of F against $^1/_E$ on the grid provided. (3 marks)



c) Use the graph to estimate the values of **a** and **b.** (3 marks)

$a \Longrightarrow y\ intercept = -2.48$

$b \Longrightarrow gradient\ of\ the\ graph = 0.5$

d) What is the relationship between F and E. (1 marks)

$F = \frac{0.5}{E} - 2.48$ OR $F = 0.5E^{-1} - 2.48$

e) Find E correct to 4 significant figures when F= 6.4 (1 marks)

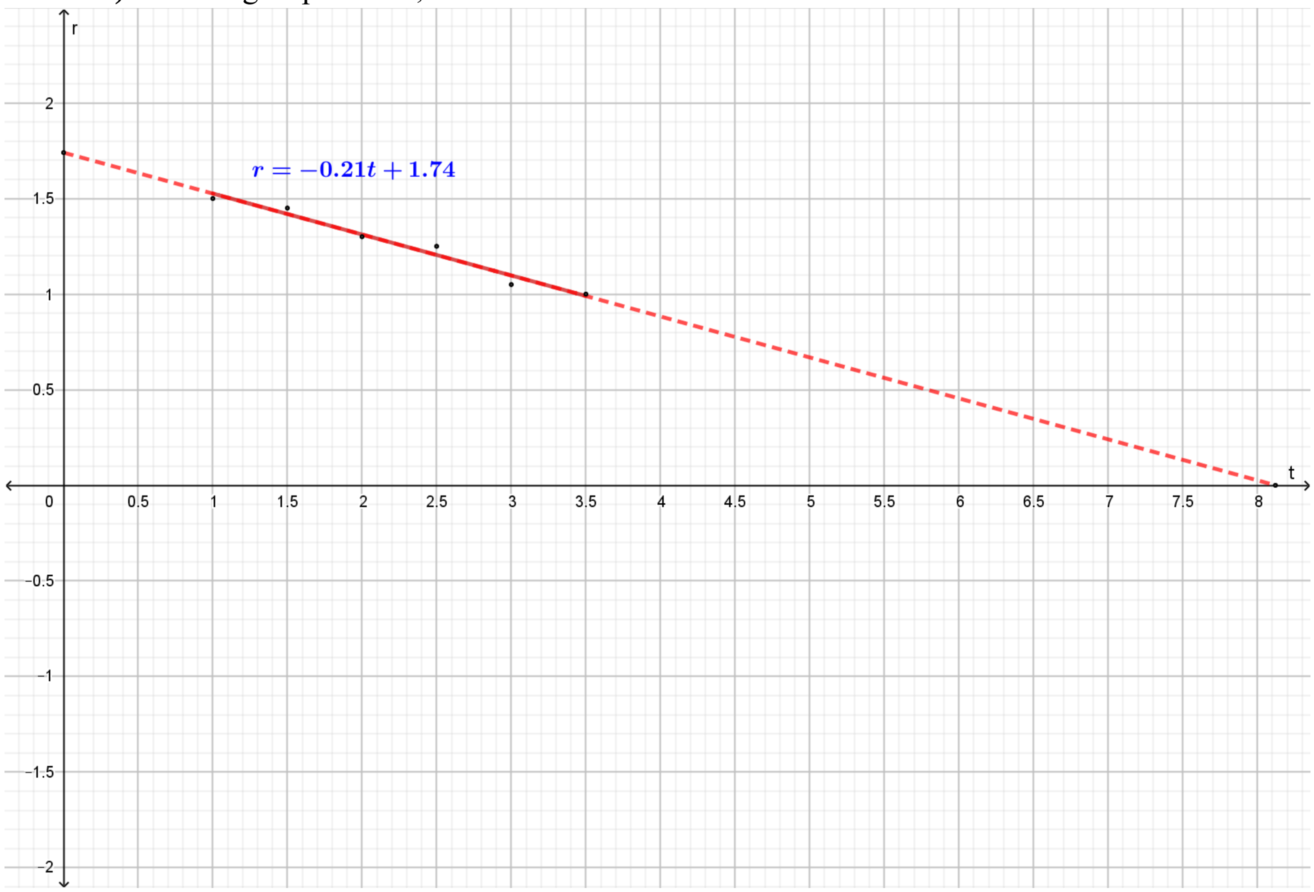
$6.4 = \frac{0.5}{E} - 2.48$

$E = \frac{0.5}{8.88} = 0.0563$

5. In an experiment involving two variables t and r, the following results were obtained.

t	1.0	1.5	2.0	2.5	3.0	3.5
r	1.50	1.45	1.30	1.25	1.05	1.00

a) On the grid provided, draw the line of best fit for the data.



b) The variables r and t are connected by the equation $r = at + k$ where **a** and **k** are constants. Determine;

i. The values of **a** and **k.**

$a \Longrightarrow gradient = -0.21$

$k \Longrightarrow y - intercept = 1.74$

ii. The equation of the line of best fit.

$r = -0.21t + 1.74$

iii. The value of t when $r = 0$.

$t = 8.12$

6. Two variables R and P are connected by a function $R = kP^n$ where k and n are constants.

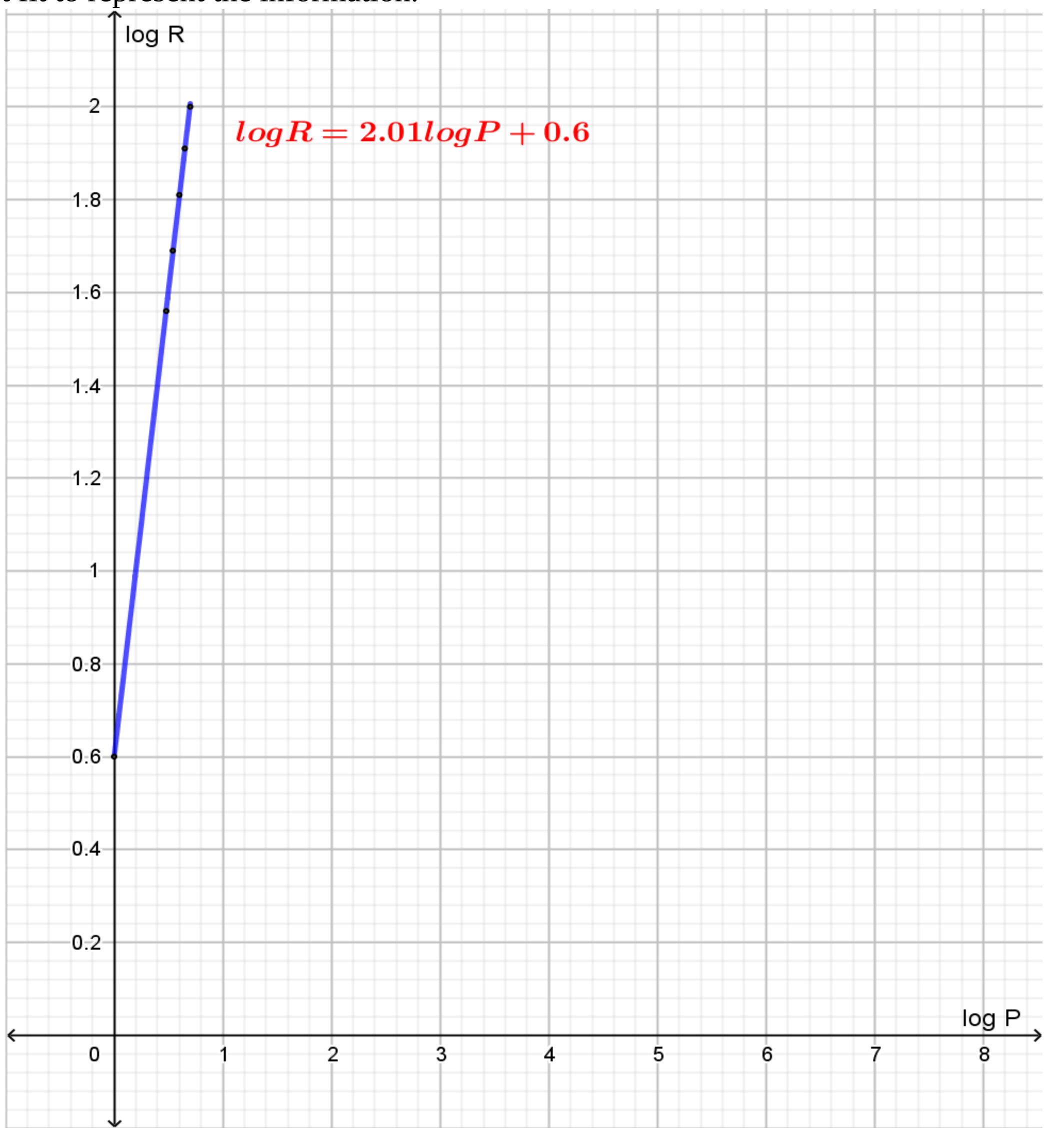
The table below shows the data involving the two variables.

P	3	3.5	4	4.5	5
R	36	49	64	81	100
$\log P$	0.48	0.54	0.60	0.65	0.70
$\log R$	1.56	1.69	1.81	1.91	2.00

(a) Complete the table above for $\log P$ and $\log R$ correct to 2 decimal places.
(b) Express $R = kP^n$ in a linear form.

$\log R = \log k + n \log P$

(c) Using a scale of 1 cm for 0.1 on the x – axis and 1 cm for 0.2 on the y – axis, draw a line of best fit to represent the information.



(d) Using your graph, determine the law connecting R and P.

$\log R = 2.01 \log P + 0.6$

7. Two quantities P and r are connected by the equation $P = ar^n$ where $\boldsymbol{a}$ and $\boldsymbol{n}$ are constants. The

table below gives the values of P and r.

P	1.2	1.5	2.0	2.5	3.5	4.5
r	1.58	2.25	3.39	4.74	7.86	11.50

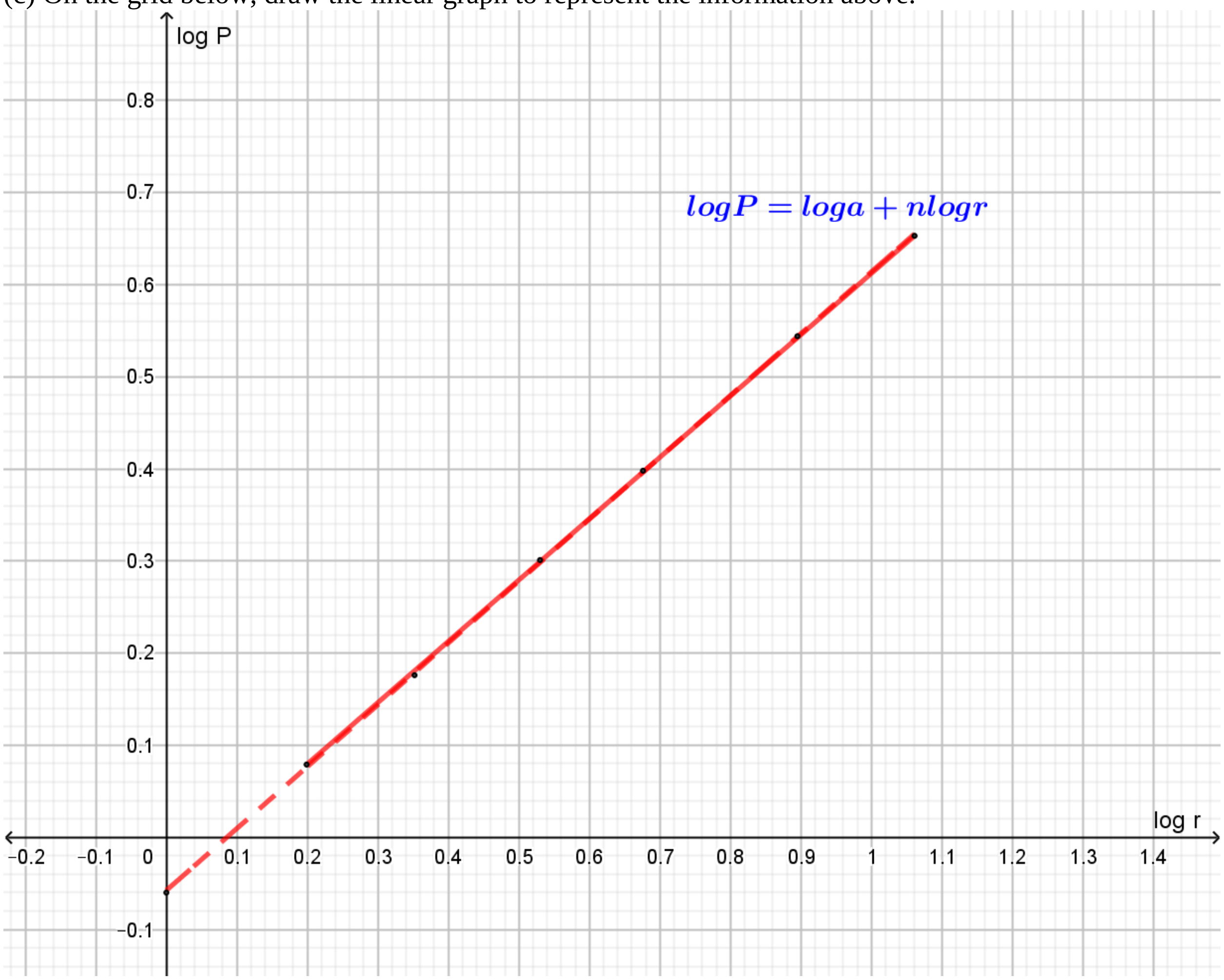
(a) Express the equation $P = ar^n$ in the form $y = mx + c$ where m and c are constants.

$\log P = \log a + nlog\ r$

(b) Complete the table above for $\log P$ and $\log r$ correct to 2 decimal places.

P	1.2	1.5	2.0	2.5	3.5	4.5
r	1.58	2.25	3.39	4.74	7.86	11.50
$\log P$	0.079	0.176	0.301	0.398	0.544	0.653
$\log r$	0.199	0.352	0.530	0.676	0.895	1.061

(c) On the grid below, draw the linear graph to represent the information above.



(d) From your graph, determine the values of the constants a and n hence find P when $r = 8$

$n = 0.67$

$a = 10^{-0.06} = 0.87$

$\log P = 0.67 \log r - 0.06$

$\log P = (0.67 \times 0.903) - 0.06 = 0.54501$

$p = 10^{0.54501} = 3.5075995 \cong 3.51$

8. The variables P and Q are connected by the equation $P = ab^Q$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants. The values of P and Q are given below.

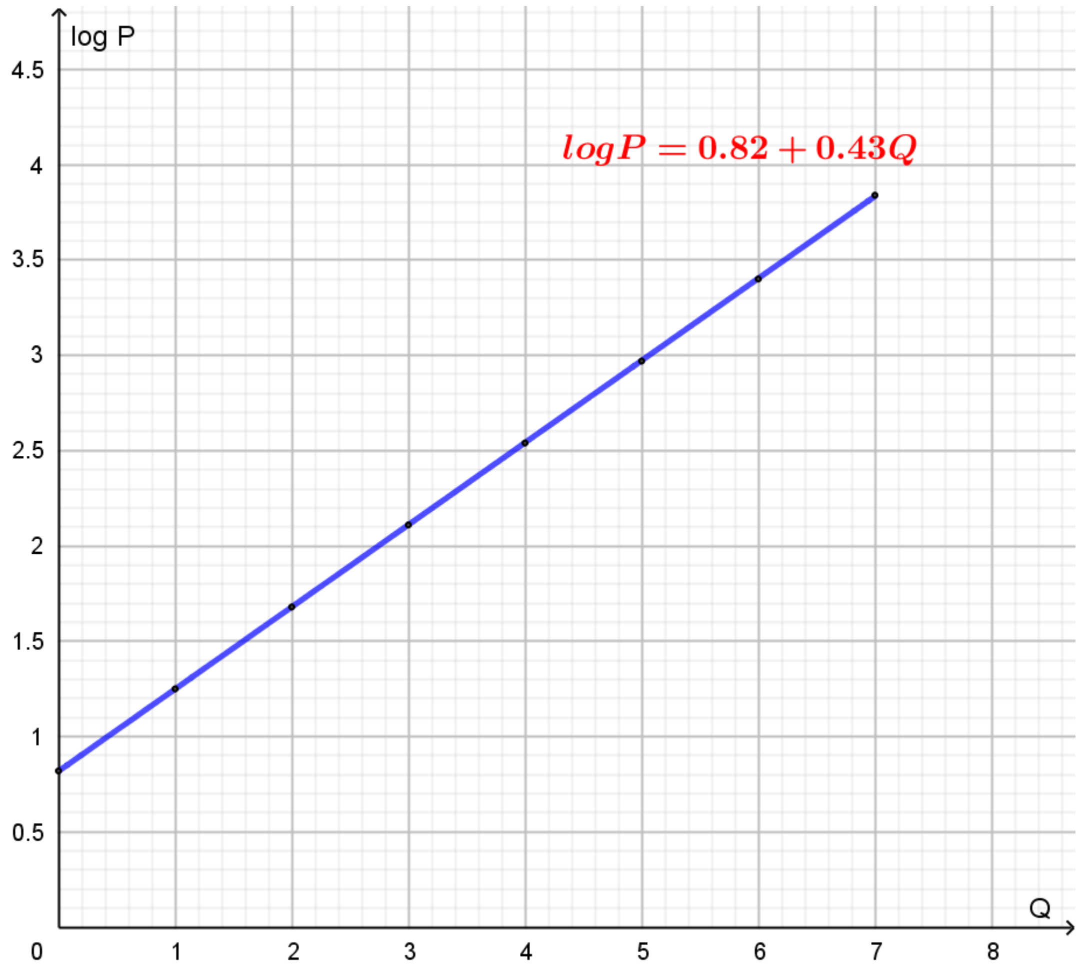
P	6.56	17.7	47.8	129	349	941	2540	6860
Q	0	1	2	3	4	5	6	7
$\log P$	0.82	1.25	1.68	2.11	2.54	2.97	3.40	3.84

(a) Complete the table for $\log P$ correct to 2 decimal places.

(b) State the equation that would give a straight line.

$\log P = \log a + Q\ \log b$

(c) On the grid provided below, draw a suitable straight line graph to represent this information. Use the scale of 1 cm for 1 unit on the horizontal axis and 1 cm for 0.5 units on the vertical axis.



(d) From your graph, determine the law connecting P and Q.

$\log P = 0.43Q + 0.82$

(e) Find P when $Q = 4.6$

$\log P = 0.82 + (0.43 \times 4.6) = 2.798$

$p = 10^{2.798} = 628.05$

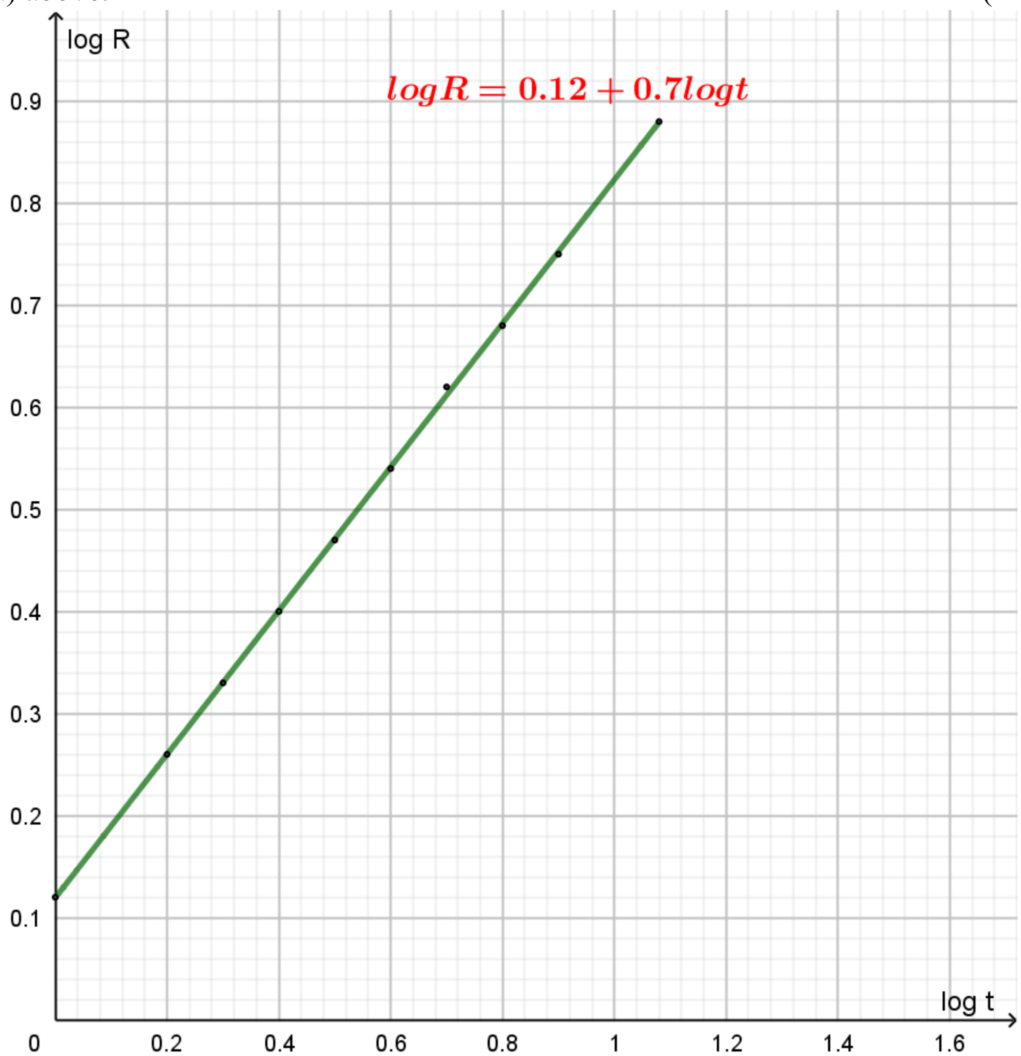
9. Two variable quantities R and t are connected by the equation $R = kt^n$ where k and n are constants. The table below gives the values of R and t.

R	1.82	2.14	2.51	2.95	3.47	4.17	4.79	5.62	7.59
t	1.58	2.0	2.51	3.16	3.98	5.01	6.31	7.94	12.0
$\log R$	0.26	0.33	0.40	0.47	0.54	0.62	0.68	0.75	0.88
$\log t$	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.08

a) Find a linear equation which connects R and t. (2 marks)

$\log R = \log k + n \log t$

b) On the graph provided, draw a suitable straight line graph to represent the relation in part (a) above. (4 marks)



c) Hence estimate to one decimal place, the values of k and n. (4 marks)

$n = gradient = 0.7$

$\log k = y\ intercept = 0.12$

$k = 10^{0.12} = 1.318 \cong 1.3$