BIOSTATISTICS LEVEL II NOTES 2014

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OUTLINE

- INTRODUCTION TO BIOSTATISTICS
- PRESENTATION OF DATA
- CALCULATING MEASURES OF DISPERSION IN GROUPED DATA

1. INTRODUCTION TO BIOSTATISTICS

BY: ERASTUS NJERU

OBJECTIVES

A. Definitions

B. Types of data

C. Descriptive statistics for quantitative & qualitative data

- Measures of central tendency & dispersion
- Data presentation (frequency tables & graphs)

A. DEFINITIONS

- Statistics:
  - This is the science concerning <u>collection</u>, <u>organization</u> & <u>summarization</u> of data as well as <u>interpretation</u> and <u>drawing</u> <u>inference</u>.
  - Statistics are the summary of the indices for data obtained from a sample.
- Biostatistics:
  - Application of statistical methods in biological sciences.
  - It is divided into 2 parts:
    1. Descriptive: organization & summarizing
    2. Inferential: drawing inference

IMPORTANCE OF BIOSTATISTICS

- The main interest of public health professionals is to improve the health status of the population through informed decision making.
- The public health professional should be able to translate data into meaningful information usable as evidence for public health decisions, ideally policies.

BIOSTATISTICS EQUIPS WITH SKILLS FOR:

- The management and analysis of health data
- Critical appraisal of available health literature and new findings
- Sharing new information with colleagues and policy makers
- Preparation of information to lobby for material support and translating knowledge to policy

KEY CONCEPTS

- Population: collection of **all** items/ subjects **of interest**
- Sample: part of the population selected to represent the population. Could be:
  - Random
  - Non – random
- Datum: raw fact/ information collected on one individual of interest, on a variable of interest.
  - Data: facts on two or more individuals

CONT.

- Parameters: summary of indices for describing the entire population
- Statistics: summary indices for data obtained from a sample
- Variables: characteristics that vary from subject to subject or from time to time.
  - Some variables change frequently while others don't change as often

TYPES OF VARIABLES

- Can be classified in accordance to the following contexts:
  - In data collection/ presentation (descriptive statistics)
    - How the values are measured/ observed (scale of measurement)
  - In data analysis (inferential statistics)
    - Whether they are a response or explanatory variable, i.e., the purpose of the variables.

THE ABOVE CAN ALSO BE CHARACTERIZED AS FOLLOWS

- Discrete variables
  - Can take only certain values and none in between
  - E.g. the number of patients in a hospital census
- Continuous variables
  - May take any value (typically between certain limits)
  - Most biomedical variables are continuous.
  - E.g. patient's weight, height, age, BP etc.

B. TYPES OF DATA: Q/Q CLASSIFICATION

QUANTITATIVE:

- The values of a quantitative variable can either be:
  - **Discrete**: characterized by gaps (interruptions) in the values that it can assume e.g. number of lesions, number of children etc.
  - **Continuous**: can assume any value on the real number line e.g. various measurements on individuals such as weight, age, gestation age etc.

QUALITATIVE:

- The values of a qualitative variable divide the population into categories e.g. gender

DATA WILL ALWAYS FORM ONE OF FOUR SCALES OF MEASUREMENT (STEVEN'S CLASSIFICATION)

NOMINAL/ CATEGORICAL e.g. gender, race, religion

- The data are in form of labels dividing the population into qualitative categories
- There is no order between the various values of the variables
- Nominal data that fall into only 2 groups are dichotomous data.

ORDINAL e.g. severity of edema, position in a race

- The data are in form of ranks
- There is meaningful order between the values of the variables
- There is no sensible arithmetic difference

INTERVAL e.g. temperature, calendar dates, social class & personality scales

- The data are usually in form of measurements on some scale
- They have order between the values of the variables
- There is a sensible arithmetic difference
- The zero is arbitrary and therefore ratios of scores are not meaningful.

RATIO e.g. age, weight, height

- Same properties as interval scale data
- The zero is absolute
- Most biomedical variables lie here
- The kelvin scale is the only ratio scale of temperature

2. PRESENTATION OF DATA

A. PICTORIAL PRESENTATION

B. NUMERICAL PRESENTATION

KEY CONCEPTS

- Methods of pictorial presentation of data:
  - Tables:
    - Frequency distribution table
    - Contingency table
  - Graphs:
    - Line graph, Bar chart, Pie chart, Histogram, Frequency polygon, Ogive, Stem - & - leaf, Box plot & Scatter diagram
- Measures of location & variability

A. PICTORIAL PRESENTATION OF DATA: FREQUENCY DISTRIBUTION TABLE

- In a frequency distribution table there is a list of all the possible values in descending order with a record of the frequency (f) of each.
- Grouped frequency distributions:
  - The individual scores are grouped with each group encompassing an equal class interval.
- Relative frequency distribution
  - Shows the percentage of all the elements that fall within each class interval.
    - $\text{Relative frequency} = \dfrac{f \text{ in that class interval}}{\text{n}} \text{ X } 100$

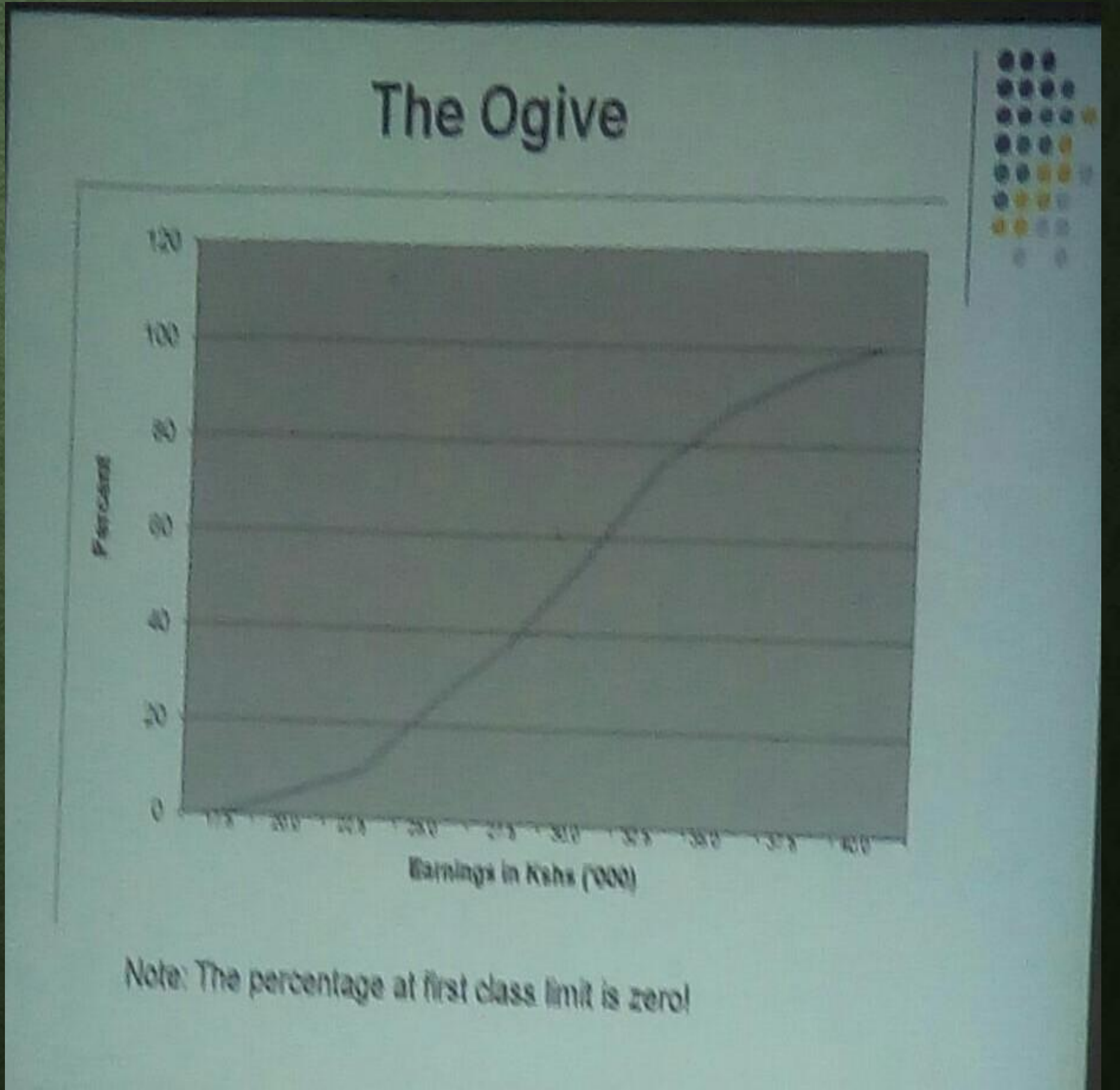
CONT.

AGE (MONTHS)	FREQUENCY	RELATIVE FREQUENCY
15 – 19	5	20%
20 – 24	6	24%
25 – 29	8	32%
30 – 34	4	16%
35 – 39	2	8%
TOTAL	25	100%

Age distribution of children seen, center X , August 2004

OGIVE (CUMULATIVE FREQUENCY POLYGON)

- This shows the percentage of elements lying within and below each class interval.
- Typically forms a characteristic S – shaped curve, i.e., ogive.



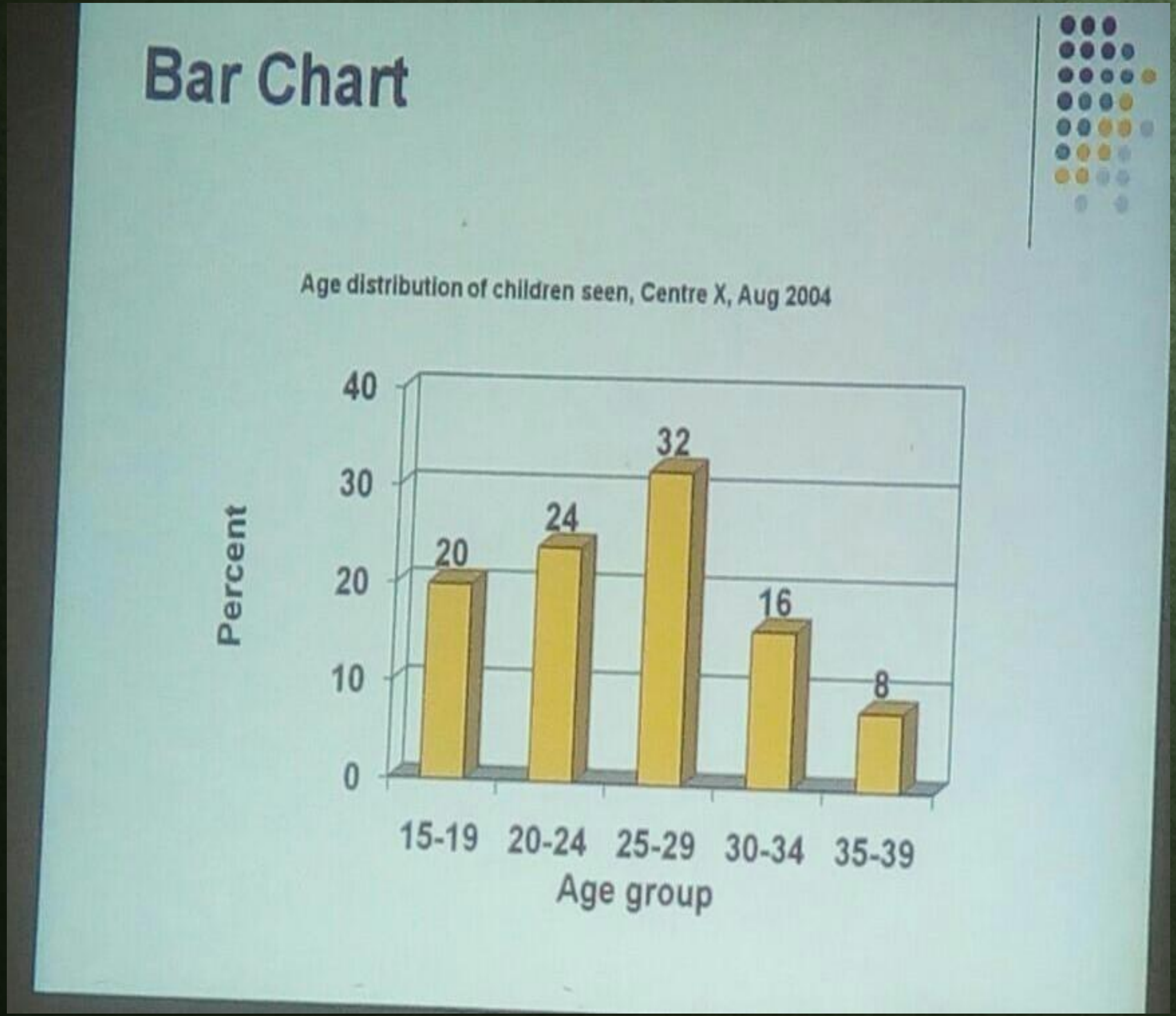
GRAPHICAL PRESENTATIONS OF FREQUENCY DISTRIBUTIONS

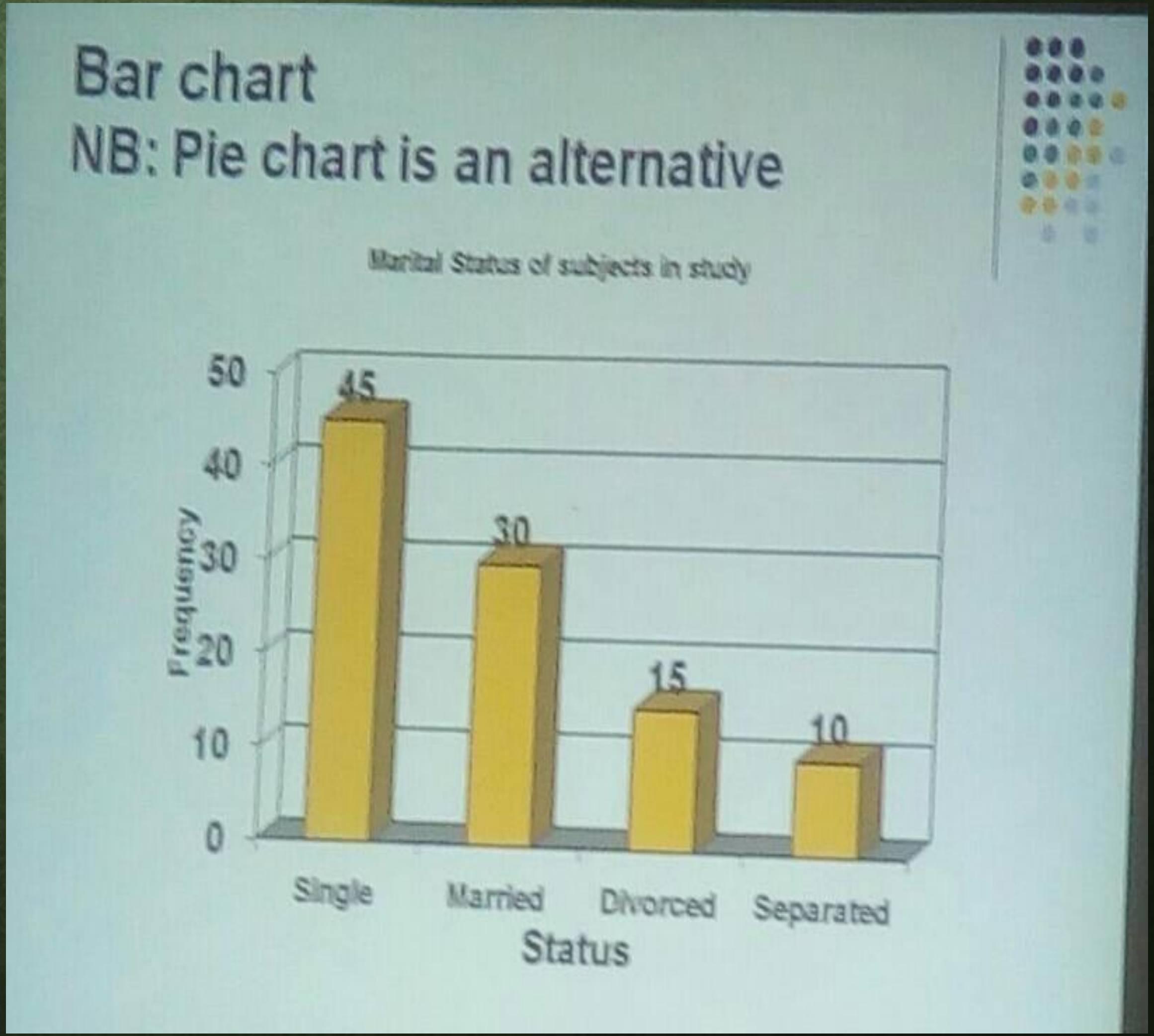
- Frequency distributions are often presented as graphs, most commonly as histograms.
- In the **histogram** the abscissa (X or horizontal axis) shows the grouped scores and the ordinate (Y or vertical axis) shows the frequencies.
- To display nominal data, a **bar graph** is typically used.
  - A bar graph is identical to frequency histograms except that each rectangle on the graph is separated from the others by a space, showing the data form discrete categories.

CONT.

- For ratio or interval scale data, a frequency distribution may be drawn as a frequency polygon in which the mid points of each class interval are joined by straight lines.

BAR & PIE CHART





HISTOGRAM
WHAT IS IMPORTANT:

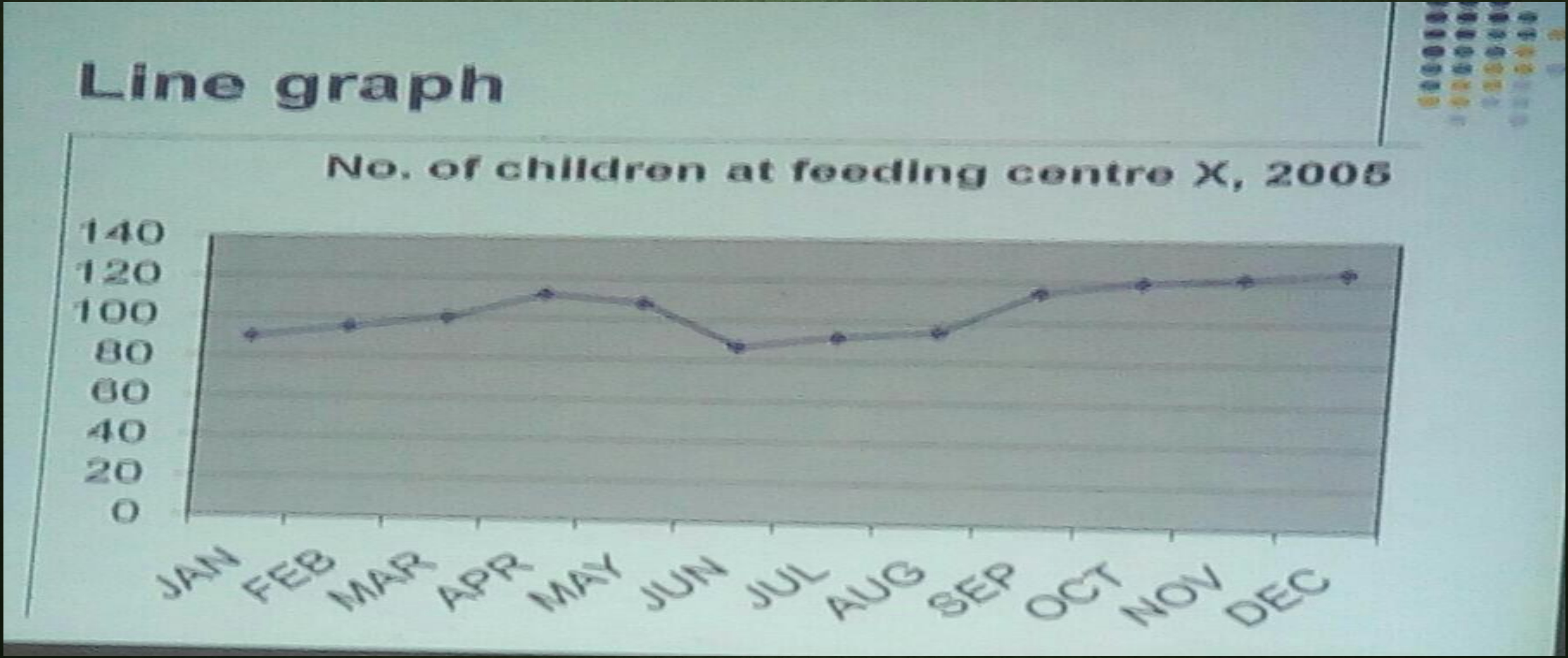
- Skewness:
  - Measure of the asymmetry (departure from symmetry) of the distribution
  - Left skewed data are characterized by a pile – up of observations to the right (long left tall)
  - Also referred to as a negative skew
  - Right skewed data are characterized by a pileup of observations to the left (long right tail)
  - Also referred to as a positive skew
- Kurtosis – degree of peakedness

CONTINGENCY TABLE

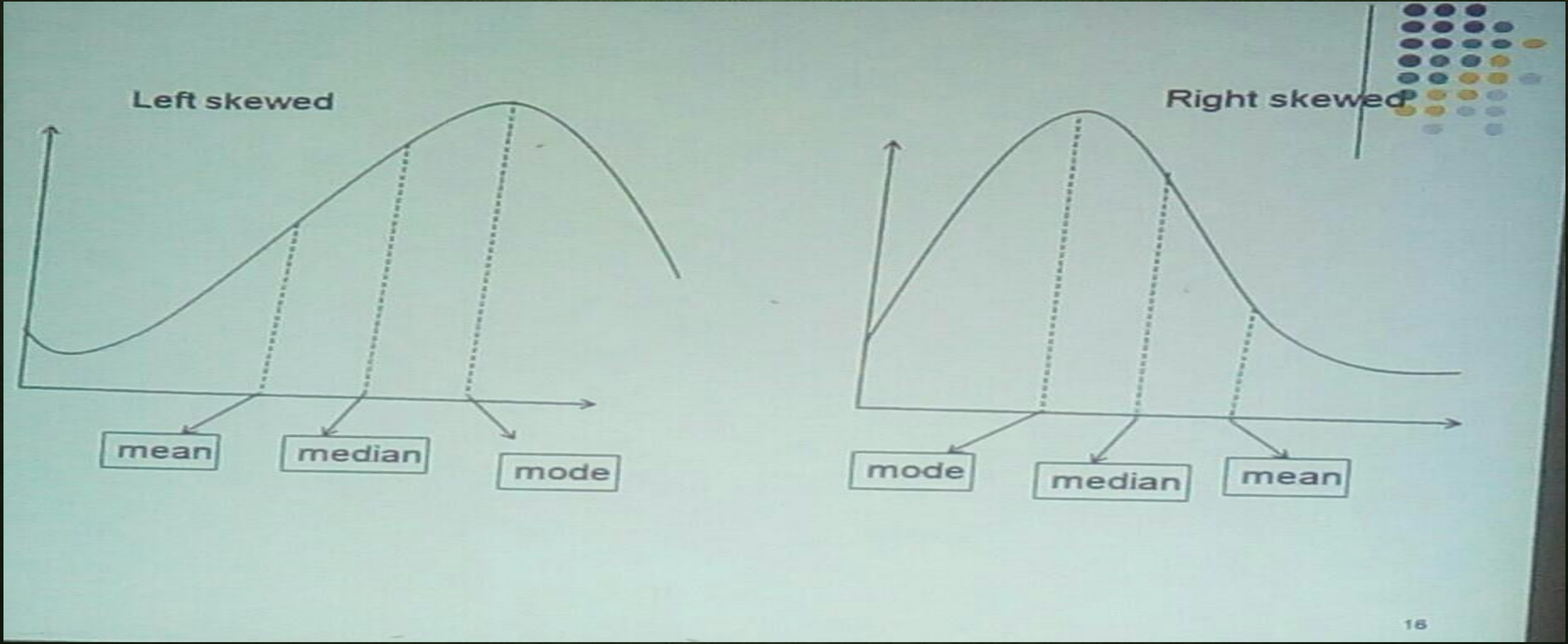
	MALE	FEMALE	TOTAL
SINGLE	9 (45%)	20 (40%)	29 (41.4%)
MARRIED	6 (30%)	25 (50%)	31
DIVORCE	3 (15%)	4 (8%)	7
SEPARATED	2 (10%)	1 (2%)	3
TOTAL	20 (100%)	50 (100%)	70 (100%)

Distribution of subjects attending clinic X by marital status & sex, Dec 2013

LINE GRAPH

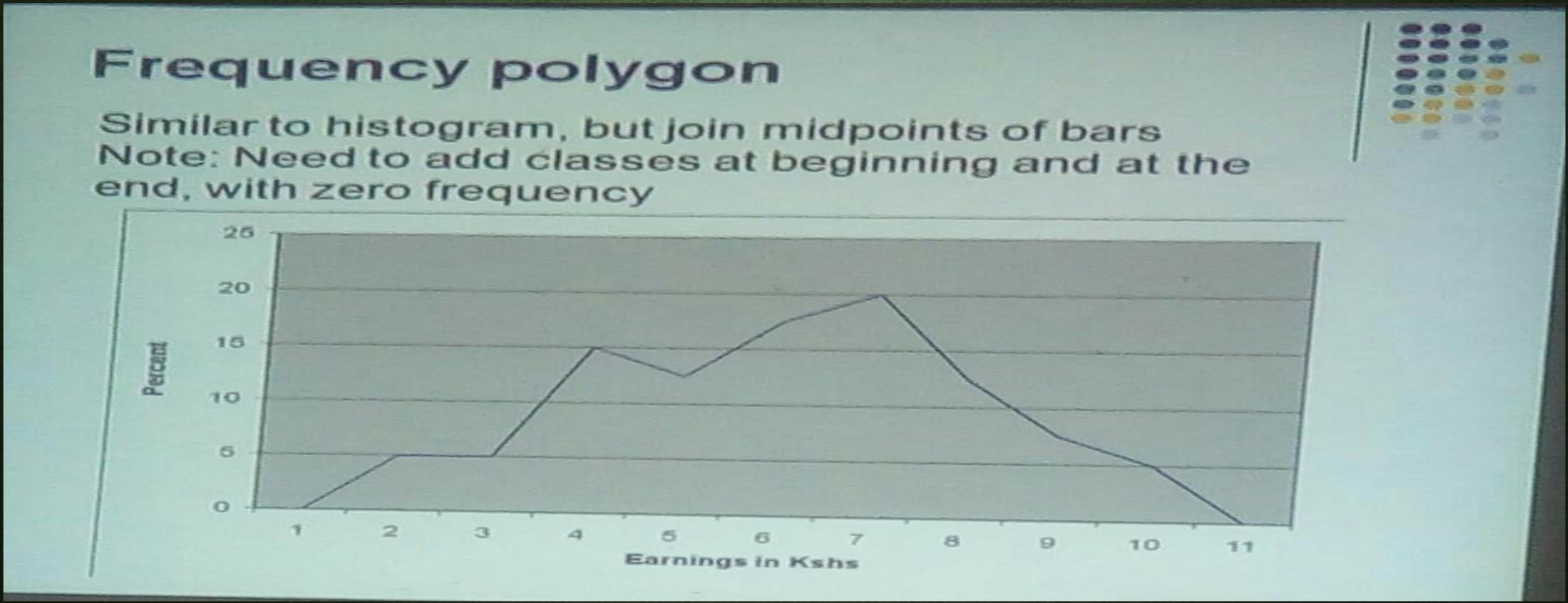

Line graph
No. of children at feeding centre X, 2005
140
120
100
80
60
40
20
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC

CONT.



Left skewed
mean
median
mode
Right skewed
mode
median
mean
16

FREQUENCY POLYGON



STEM - & - LEAF PLOTS

- Use digits in observation to give picture
- 1st set of digits → stem
- 2nd set → leaves
- Age of children example:
  - Data have only 2 digits:

Stem	Leaf
1	97
2	8625
3	20

PERCENTILES

- The p^{th} percentile is a value such that p% of the observations fall below that value.
- Example 1: 10% of data are less than the 10^{th} percentile
- Example 2: 50^{th} percentile = ?
  - Q1 = lower quartile (25% percentile)
  - Q3 = Upper quartile (75% percentile)

B. NUMERICAL PRESENTATION OF DATA

- 2 aspects of data are important in presentation/ description:
  - Location
    - On the scale of observations, where do our observations lie?
    - Also known as **central tendency or average**
  - Variability
    - How far are the observations from one another?
    - Also known as **spread or dispersion**
    - This is important ins statistical inference.

I. MEASURES OF LOCATION/ CENTRAL TENDENCY: ARITHMETIC MEAN

- Sum of all observations divided by the number of observations.
- It is symbolized by a $\{\bar{x}\}$ in the sample and a $\{\mu\}$ in the population.
- It is most amenable to mathematical manipulations and is not suitable when there are outliers, i.e., in very skewed distributions
- It is the measure of central tendency that best resists the influence of fluctuation between different samples.

FORMULA FOR

Sample mean, $\bar{x} = \frac{\Sigma x_{\text{i}}}{n}$

Population mean, $\mu = \frac{\Sigma x_{\text{i}}}{N}$

MEDIAN

- This is the figure that divides the frequency distribution in half when all scores are listed in order.
- If the number of observation is even, calculate the arithmetic mean of the 2 middle observations to arrive at the median.
- The median is insensitive to small numbers of extreme scores (outliers) and therefore, it is a very useful measure of central tendency for highly skewed distributions.
- It is the same as the 50^{th} percentile.

MODE

- This is the observed value that occurs with the greatest frequency.
- On a frequency polygon, it is the highest point on the curve.
- If 2 scores both occur with the greatest frequency, the distribution is bimodal; if more than 2 scores occur with the greatest frequency, the distribution is multimodal.
- It is not unique

II. MEASURES OF VARIABILITY/ DISPERSION/ SPREAD

- Variability is the extent to which the observations are clustered together or scattered about.
- There are 3 important measures of variability:
  - Range
  - Variance
  - Standard deviation

RANGE

- This is the difference between the largest and smallest value.
- Not suitable when there are outliers
- Mid – range = $\frac{Q_3 + Q_1}{2}$
- Inter – quartile range (IQR) = $Q_3 – Q_1$

VARIANCE & STANDARD DEVIATION

- Sample variance is defined as the sums of squares of the differences between each observation in the sample and the sample mean divided by 1 less than the number of observations.

CONT.

- Calculation of both the variance & the standard deviation, involves the use of **deviation scores**, which are found <u>by subtracting the distribution's mean from each value</u>.
  - Deviation score, $x = X - \bar{x}$
    - Where {X} is a value & {$\bar{x}$} is the mean
    - NB: the sum of all deviation scores {Σx} should be zero.
- Variance of a distribution is simply the mean of the squares of all the deviation scores in the distribution.

FORMULA FOR VARIANCE

1. Find deviation score (x) for each value
2. Square each deviation score to eliminate negative signs
3. Obtain their mean

- NB: (n – 1) is the denominator for sample variance, instead of n. This is done as the former gives a <u>less biased estimate</u> of variance of the population.

Population variance, σ^2

$$= \frac{\Sigma\,(X - \mu)2}{N} \text{ or } \frac{\Sigma\,(x)^2}{N}$$

Sample variance, S^2

$$= \frac{\Sigma\,(X - \bar{x})2}{n - 1} \text{ or } \frac{\Sigma\,(x)2}{n - 1}$$

STANDARD DEVIATION

- That variance is expressed in square units of measurement, limits its usefulness as a descriptive term and hence the standard deviation remedies this problem.
- It is the **square root of variance** and so it is expressed in the same units of measurement as the original data.
- The standard deviation of a population is expressed as, $\{\sigma\}$ and that of a sample is expressed as $\{S\}$

CONT.

- The SD is particularly useful in normal distributions because the proportion of values/ elements in the normal distribution (i.e., the proportion of the area under the curve) is a constant for a given number of SDs above or below the mean.
  - Approximately 68.27% of the distribution falls within ± 1 SD from the mean.
  - **Approximately 95% of the distribution falls within ± 0.96 SD from the mean.**
  - Approximately 95.45% of the distribution falls within ± 2 SD from the mean.
  - Approximately 99% of the distribution falls within ± 2.58 SD from the mean.
  - Approximately 99.73% of the distribution falls within ± 3 SD from the mean.

COEFFICIENT OF VARIATION

- Standard deviation divided by mean
  - $\frac{S}{\bar{x}}$
- It has no units and can be used to compare variability in 2 groups/ populations where different units are used.

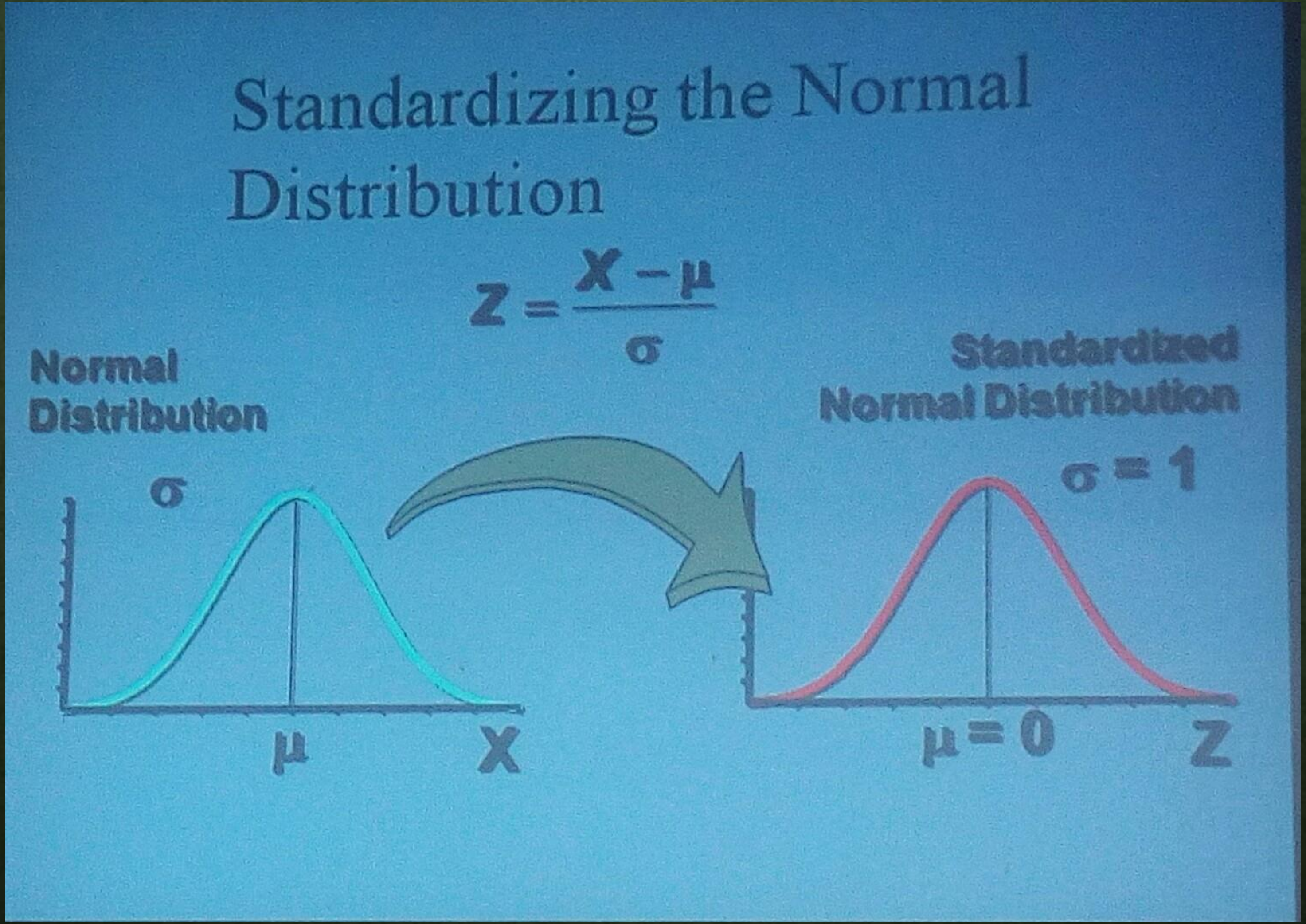
Z SCORES

- The **z score/ standard normal deviate** of any statistical observation in a normal distribution is the number of standard deviations the observation lies above or below the mean of the distribution,
- If the observation lies above the mean it will have a positive z score and if it lies below the mean it will have a negative z score
  - Z score, $Z_i = \dfrac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \dfrac{X_i - \mu}{\sigma}$
- i.e. for every observation X_i subtract population mean and divide by the standard deviation.

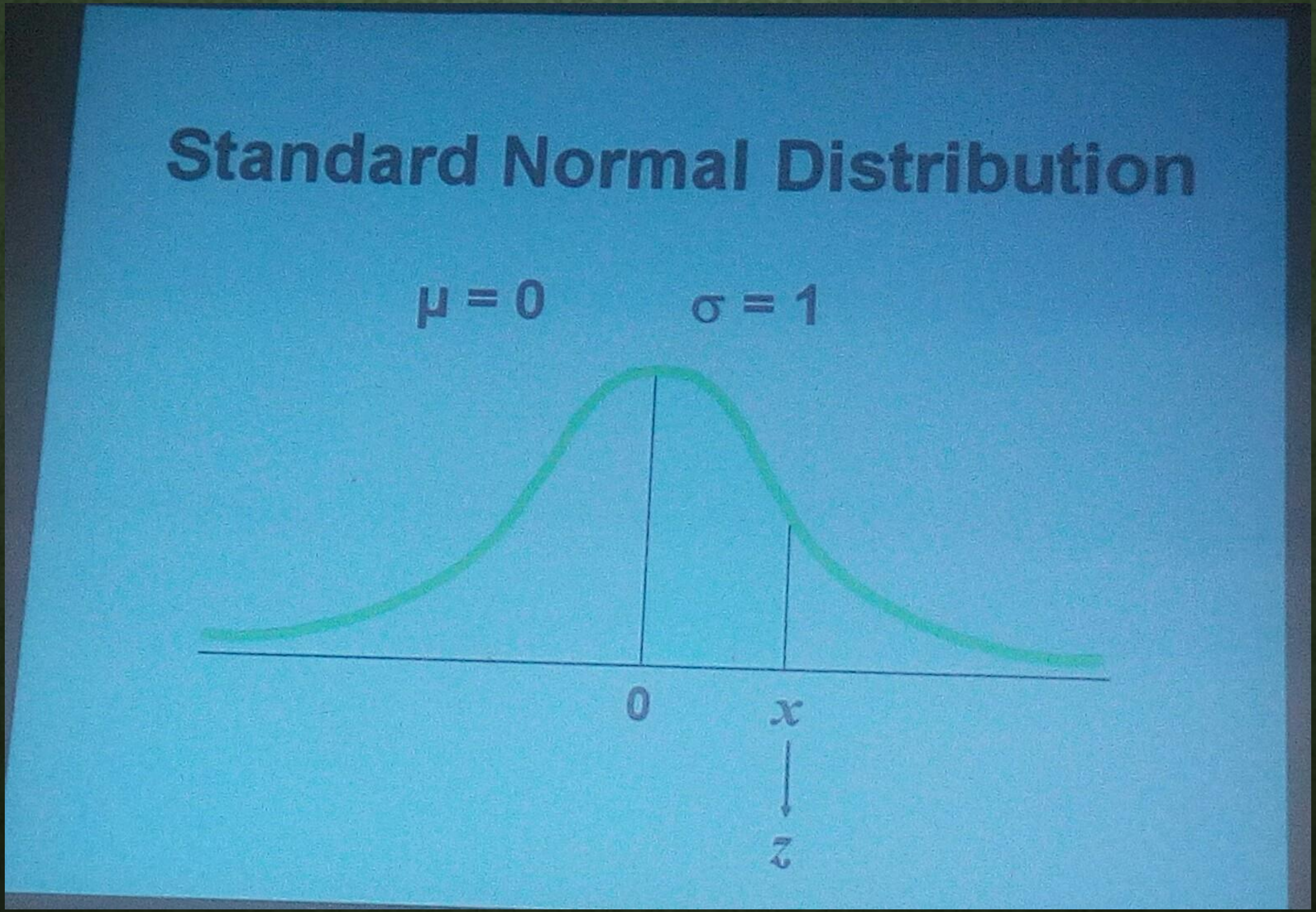
USE OF Z SCORES

- Since they are standardized or normalized they allow scores on different normal distributions to be compared e.g. a person's height and his weight
- They can be used to find the proportion of a distribution that corresponds to a particular score.
- They can be used to find the score that divides the distribution into specified proportions
- They allow the specification of the probability that a randomly picked observation will lie below or above a particular score.

STANDARDIZING THE NORMAL DISTRIBUTION



STANDARD NORMAL DISTRIBUTION



FINDING PROBABILITIES FOR Z SCORES

Standard Normal (z) Probabilities (Above/below)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010

- Body of table contains $P(Z \leq z^*)$.
- Left-most column of table shows algebraic sign, digit before the decimal place, the first decimal place for z^*.
- Second decimal place of z^* is in column heading.
- The values in the body of Table (next) refer to the region **under the curve.**

EXAMPLE

- The population of neonates in a certain hospital is known to have birth weight that are normally distributed with mean 3.2 kg and variance 2.5 kg
  - What is the probability of getting a neonate with a birth weight of > 3.6kg?
  - What is the probability of getting a neonate with a birth weight between 2.0 and 2.5 kg?

BERNOULLI TRIALS

- This is an experiment (usually observation of an individual) which has two possible outcomes, i.e.,:
  - Y/N; 0/1; +ve or –ve; present/ absent; success/ failure
- Examples:
  - Is the baby a boy?
  - Does the patient have cholera?
  - Is the client positive?
  - Does the toss of coin result in ‘head’?

BINOMIAL DISTRIBUTION

- Note that it is possible to have a Bernoulli process from a non – binary variable.
  - Ex. Does patient have severe edema? Does roll of die result in a ‘2’ ?
- A series of Bernoulli trials (which can be viewed as a series of observations) gives rises to a binomial distribution.

3. CALCULATION OF STATISTICS FROM GROUPED DATA

BY: ERASTUS NJERU

THE CODED DATA METHOD

1. Calculate the cumulative frequency & estimate the group containing the median. Assign it the value 0.
2. To get the coded data, x_c =
$$\frac{\text{Mid point of group} - \text{Mid point of median group}}{\text{Class Width}}$$
3. Get the coded mean, $\bar{x}_c = \frac{\Sigma f \text{xc}}{\Sigma f}$
4. Get the coded variance, $S_c{}^2 = \frac{\Sigma f\{x_c{}^2\} - n\{\bar{x}_c\}^2}{N-1}$
5. Reverse the coding

REVERSING THE CODING

- To get the mean, variance & standard deviation of the data set, consider the following:
- If a constant $\boldsymbol{k}$, is added to all observations:
  - Mean increases by $\boldsymbol{k}$
  - Variance doesn't change
- If all observations are multiplied by a constant $\boldsymbol{k}$:
  - Mean & Standard deviation are each multiplied by $\boldsymbol{k}$
  - Variance is multiplied by $\boldsymbol{k}^2$

THEREFORE:

- Actual mean = {Coded mean, $\bar{x}_c$ X Class Width} + Mid point of median group
- Actual variance = Coded variance X {Class Width}^{2}
- Actual standard deviation = Coded SD X Class Width

Class limits	Mid – point, (x_i)	Freq. (f)	c. f	Coded data (x_c)	fx_c	$f\{x_c\}^2$
15 - 20	17.5	55	55	-2	-110	220
20 - 25	22.5	69	124	-1	-69	69
25 - 30	27.5	84	208	0	0	0
30 - 35	32.5	47	255	1	47	47
35 - 40	37.5	26	281	2	52	104
					-80	440

- Coded mean, $\bar{x}_c = {}^{\Sigma f xc}/_{\Sigma f}$
- ${}^{-80}/_{281}$
- $\boxed{-0.2847}$

- Coded variance, $S_c^2 = {}^{\Sigma f\{x_c^2\} - n\{\bar{x}_c\}^2}/_{N-1}$
  - ✓ $\Sigma f\{x_c\}^2 \rightarrow 440$
  - ✓ $n(\bar{x}_c)^2 = 281 \text{ X } \{-0.2847\}^2 \rightarrow 22.78$
- ${}^{417.22}/_{280} \rightarrow \boxed{1.4901}$

- Actual Mean = {(- 0.2847) X 5} + 27.5
- ➔ $\boxed{26.0765}$

- Actual variance = 1.4901 X 5^2
- ➔ $\boxed{37.2525}$
- Actual standard deviation = $\sqrt{(1.4901)}$ X 5
- ➔ $\boxed{6.1035}$

CALCULATING THE MEDIAN FOR GROUPED DATA

1. Find the n^{th} value of median = $\frac{c.\ f}{2}$

2. Median =

- Lower Median Class Boundary (l.m.c.b) +

$$\left\{\frac{n^{th}\text{ value of median} - c.\ f\text{ of previous class group}}{\text{Frequency of median class group}}\right\} \text{X Class width}$$

Weight in kg for a sample of school children

Class limits	Mid - point, (x_i)	Freq., (f)	fx	$\{fx\}^2$	c. f
15 – 20	17.5	5	87.5	1531.25	5
20 – 25	22.5	6	135	3037.5	11
25 – 30	27.5	8	220	6050	19
30 – 35	32.5	4	130	4225	23
35 – 40	37.5	2	75	2812.5	25
		25	647.5	17656.25	

For calculating **mean** the assumption is that all observations in each interval lie at the mid – point of the interval:

- Mean = $\frac{\Sigma fx}{\Sigma f}$
- $\frac{647.5}{25} \rightarrow \boxed{25.90}$

- Variance = $\frac{\Sigma f\{x^2\} - n\{x\}^2}{N - 1}$
- $\frac{17656.25 - 25\ \{25.90\}^2}{24}$
- $\frac{886}{24} \rightarrow \boxed{36.92}$

For calculating the median the assumption is that all observations in the interval containing the median are equidistant:

- $\frac{25}{2}$ = 12.5th value (**corresponds to the class group whose limits are 25 – 30**)
- $24.5 + \{\frac{12.5 - 11}{8} \text{ X } 5\} = 24.5 + 0.9375 = \boxed{25.4375}$

"IT IS NO MEASURE OF HEALTH TO BE WELL ADJUSTED TO A PROFOUNDLY SICK SOCIETY"

JESUS CHRIST IS THE ONLY WAY, THE ONLY TRUTH & THE ONLY LIFE.